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THEORETICAL THERMAL-ACTED HEAT PUMP

BY
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Abstract. The theoretical Vuilleumier machine is defined, inside which the gas evolves in simple thermodynamic processes. Thus, the theoretical Vuilleumier machine achieves the greatest possible performances. Equal lengths of the functional phases are imposed by the condition that between the movement laws of the displacers a phase angle γ exist. Using the properties of the theoretical Vuilleumier machine, the optimum cylinder diameter ratio (for which the heats exchanged are maximum) is deduced.

Keywords: theoretical thermal-acted heat pump, Vuilleumier, diameter ratio.

1. Introduction

A thermal-acted heat pump (a Vuilleumier machine, VM or TAHP-V) [1], [2], [3] is a machine inside which a constant amount of gas evolves inside an almost constant total volume. The gas lies inside several heat exchangers and four variable volume chambers placed (most often) inside two cylinders, each cylinder being fit with its own displacer piston. There are three levels of temperature inside the machine. The machine receives heat at the lower temperature of the cycle by expanding the gas inside the low temperature chamber. Pressure variation inside the machine is acquired by heating the agent inside a high temperature chamber and by cooling the agent inside two intermediate temperature chambers.

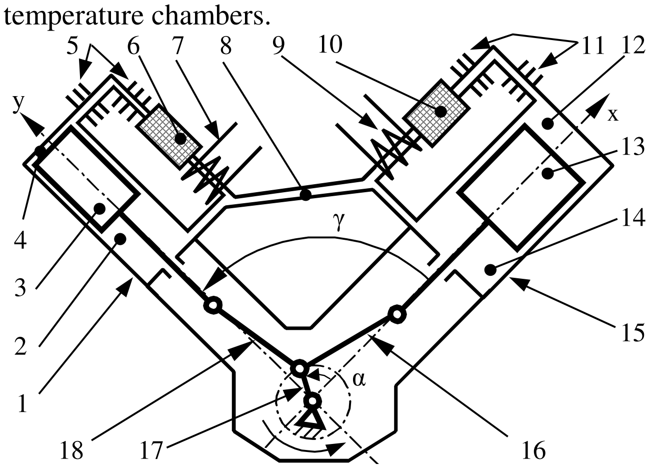

Fig. 1.- Vuilleumier machine.

Accordingly to the schematic diagram in fig. 1 [4], a Vuilleumier machine is comprised of a cold cylinder 1 and a hot cylinder 15 inside which the cold displacer 3 and the hot displacer 13 work. The cold cylinder and displacer share a diameter inferior to the one shared by the hot cylinder and displacer. A drive comprised of crankshaft 17 and rods 16 and 18 provide movement for the displacers. The cold displacer splits the space inside its cylinder in two chambers: a low temperature one 4 and an intermediate temperature one 2. Inside the hot cylinder the hot displacer delimitates a high temperature chamber 12 and an intermediate temperature chamber 14. Each cylinder is fit with its own heat exchanger set. The cold cylinder has a low temperature heater 5, a low temperature regenerator 6 and an intermediate temperature cooler 7. The hot cylinder is fit with an intermediate temperature cooler 9, a high temperature regenerator 10 and a high temperature heater 11. The intermediate temperature cooling chambers are connected through pipe 8.

2. Defining the Vuilleumier Theoretical Machine

The paper defines the theoretical VM and analyzes the way in which the cycle is realized in this machine. The thermal acting of the theoretical VM implies that the machine must not and does not have to exchange work, so the total volume occupied by the agent is constant. The lack of any work exchanged with the exterior imposes that the working agent be the perfect gas. So, the instan-taneous pressure is identical in the entire VM. The VM must receive heat from the cold and hot sources and concede heat to the intermediate temperature source.

In order the gas inside the low temperature chamber volume V_{lt} to absorb heat from the cold source, the pressure inside the machine must drop during this process. The pressure drop is obtained as result of the hot displacer movement from BDP to TDP, that is by displacing an amount of gas from the maximum temperature chamber into lower temperature chambers. In order that the maximum possible fraction of the total amount of gas to be found inside V_{lt}, the volume of the heat exchangers must be null and volume V_{lt} constant and maximum. The role of the heat exchangers is played by portions of the cylinder walls. The regenerators are ideal (regeneration efficiencies being 100%) and are also represented by portions of the cylinder walls (or by piston regions).

Maintaining volume V_{lt} constant during the heat absorbtion process imposes that the cold displacer be meanwhile halted in its BDP. It results that the maximum of volume V_{ht} and the maximum of volume V_{it2} are equal. The stem diameters must be null. It results that for the theoretical VM only four functional spaces are needed, each having variable volume.

Heat extraction from the cold cylinder intermediate temperature chamber of V_{it1} volume is achieved as the hot displacer moves from TDP to BDP (a fraction of the gas passing to a higher temperature, by entering inside the high temperature chamber of volume V_{ht}), increasing the pressure inside the machine. The presence of a maximum amount of gas inside V_{it1} imposes that the cold displacer be halted in its TDP, hence this piston must have two halting intervals along an entire cycle.

As the cold displacer moves from BDP to TDP, the agent inside V_{lt} passes toward higher temperature chambers and the pressure rises. The gas inside V_{it2} rejects heat and $V_{it2} = V_{it1max}$.The hot displacer is halted in BDP. Similarly, the hot displacer must halt at TDP when the cold displacer moves from TDP to BDP.

The theoretical VM must function at maximum efficiency, i.e. the heat exchanges with the exterior must be reversible. That imposes that the heat sources be thermal reservoirs of infinite capacity. In this case, in all machine chambers only constant temperature processes take place. The Vuilleumier machine fulfiling the above conditions is presented in fig. 2.

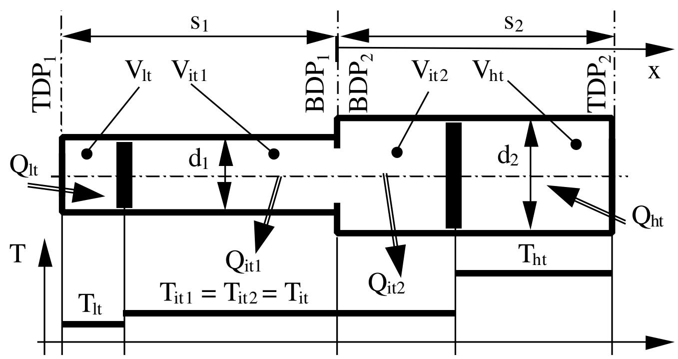

Fig. 2.- Schematic diagram of the theoretical Vuilleumier machine.

Because each displacer moves only when the other rests, the pressure variation law inside the VM as function of volume depends on displacer position and not on their particular law of displacement. As result, the length of the functional phases (during each of which one displacer moves while the other rests) is indifferent. The sum of the four phase lengths must be equal to the cycle period.

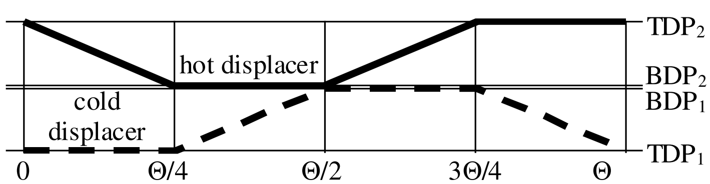

Fig. 3.- Displacer movement laws for theoretical Vuilleumier machine.

Functional phase succession shows that the cold displacer movement repeats the hot displacer movement sequence (displacement – halt – displacement– halt). The simplest solution of driving the pistons implies that the same type of mechanism be employed. The displacer movement laws are identical but phased with a fraction γ of the cycle period. The functional phase lengths are identical.

The phase angle $\gamma = \Theta/4$ assures that the displacers are never halted simultaneously (a useless situation) and never move together (in this case, the gas repartition inside VM chambers would not respect the imposed conditions). The simplest case of the linear movement of the displacers is presented in fig. 3.

3. Theoretical Vuilleumier Machine Performances

The variation of the instantaneous pressure inside the theoretical machine is obtained from the equation of the conservation of the mass inside the machine:

$$m_T = m_{lt} + m_{it1} + m_{it2} + m_{ht} = \frac{pV_{lt}}{RT_{lt}} + \frac{pV_{it1}}{RT_{it}} + \frac{pV_{it2}}{RT_{it}} + \frac{pV_{ht}}{RT_{ht}} \qquad (1)$$

and has the following expression

$$p = \frac{m_T R}{\frac{V_{lt}}{T_{lt}} + \frac{V_{it1}}{T_{it}} + \frac{V_{it2}}{T_{it}} + \frac{V_{ht}}{T_{ht}}}, \qquad (2)$$

where R is the individual gas constant and the subscript T means total.

The heats exchanged cyclically inside the machine chambers:

$$Q = L = \oint p\, dV . \qquad (3)$$

From the first law of thermodynamics we obtain that, for each chamber, during one complete cycle, the heat exchanged is equal to the work exchanged. The heats Q_{it1} and Q_{it2} are removed from the cycle, so their values are negative.

The coefficient of performance of a refrigerator VM is

$$\varepsilon_r = Q_{lt} / Q_{ht} . \qquad (4)$$

and of a Vuilleumier heat pump is

$$\varepsilon_{hp} = \left| Q_{it1} + Q_{it2} \right| / Q_{ht} = 1 + \varepsilon_r . \qquad (5)$$

4. Closed Form Solutions for Theoretical VM Performances

The heat exchanged and the coefficients of performance can be stated as closed form relations, given by the integrals from Eq. (3).

The pressure variation law is independent from the displacer movement laws. So, the performances of the theoretical VM do not depend on these laws. The integrals from Eq. (3) can be calculated for any displacer movement laws (volume variation laws). The simplest law that can be used is the linear one, that leads to the expressions in Table 1.

Table 1

Variation laws of volumes V_{ht} and V_{it2}.

	$t\in\left[0,\frac{\Theta}{4}\right)$	$t\in\left[\frac{\Theta}{4},2\frac{\Theta}{4}\right)$	$t\in\left[2\frac{\Theta}{4},3\frac{\Theta}{4}\right)$	$t\in\left[3\frac{\Theta}{4},\Theta\right)$
$V_{ht}(t)$	$V_{ht\,max}\frac{t}{\Theta/4}$	V_{htmax}	$V_{ht\,max}\left(1-\frac{t-\Theta/2}{\Theta/4}\right)$	0
$V_{it2}(t)$	$V_{ht\,max}\left(1-\frac{t}{\Theta/4}\right)$	0	$V_{ht\,max}\frac{t-\Theta/2}{\Theta/4}$	V_{htmax}

The volume variation laws for the cold cylinder chambers are phased with an angle $\gamma = \Theta/4$:

$$V_{lt}(t)=\varepsilon_v V_{ht}(t-\gamma)=\varepsilon_v V_{ht}(t-\Theta/2); \tag{6}$$

$$V_{it1}(t)=\varepsilon_v V_{it2}(t-\gamma)=\varepsilon_v V_{it2}(t-\Theta/2), \tag{7}$$

where

$$\varepsilon_v = V_{lt\,max}/V_{ht\,max} \tag{8}$$

is the ratio of maximum volumes inside cold and hot cylinders.

The following temperature ratios are defined:

$$\tau=\frac{T_{lt}}{T_{ht}}=\frac{T_{h1}}{T_{h2}} \quad \text{and} \quad \sigma=\frac{T_{lt}}{T_{it}}=\frac{T_{h1}}{T_k}. \tag{9}$$

The temperature of the gas inside the low temperature chamber T_{lt} and the temperature of the cold source T_{h1} are equal. The temperature of the gas inside the high temperature chamber T_{ht} and the temperature of the hot source T_{h2} are equal. The temperature of the gas inside intermediate temperature chambers T_{it} and the temperature of the third heat source T_k are equal.

Introducing the volume variations in Eq. (3), the heat cyclically exchanged in the hot temperature chamber takes the following closed form:

$$Q_{ht}=\oint p\,dV_{ht}=\frac{m_T\,RT_{h1}}{\sigma-\tau}\,ln\left(\frac{\sigma\varepsilon_v+\sigma}{\sigma\varepsilon_v+\tau}\,\frac{(\tau+\varepsilon_v)}{(\sigma+\varepsilon_v)}\right). \tag{10}$$

Similarly, the heat ciclically exchanged in the low temperature chamber:

$$Q_{lt}=\frac{m_T\,RT_{h1}}{1-\sigma}\,ln\left(\frac{\sigma\varepsilon_v+\sigma}{\sigma\varepsilon_v+\tau}\,\frac{(\tau+\varepsilon_v)}{(\sigma+\varepsilon_v)}\right). \tag{11}$$

Immediately the following relations are obtained

$$Q_{it2}=-Q_{ht} \quad \text{and} \quad Q_{it1}=-Q_{lt}. \tag{12}$$

The heats cyclically exchanged by the VM depend on the heat source temperatures (through T_{h1}, τ and σ), the ε_v ratio of the maximum volumes of the chambers inside cold and hot cylinders, the working agent properties (R) and on the gas mass m_T inside the machine.

The coefficient of performance of a theoretical Vuilleumier refrigerator is

$$\varepsilon_r = Q_{lt} / Q_{ht} = (\sigma - \tau)/(1 - \sigma) \tag{13}$$

and of a theoretical Vuilleumier heat pump is

$$\varepsilon_{hp} = \frac{|Q_{it1} + Q_{it2}|}{Q_{ht}} = 1 + \varepsilon_r = \frac{1 - \tau}{1 - \sigma}. \tag{14}$$

The Eq. (13) and (14) show that the COP's depend on the heat source temperatures only. The same conclusion was obtained for a VM calculated with an isothermal method [5]. The dependence is only on temperatures due to the isothermal heat exchange and due to the ideal regenerators (just as it happens at Stirling machines calculated with isothermal models, when the efficiencies depends only on heat source temperatures [1]). Efficiencies depending on other factors besides temperatures can be obtained only with physico-mathematical models more complex than the isothermal ones (e.g. semi-adiabatic model [4]).

5. Volume optimum ratio ε_v

The analysis of the Eq. (10), (11) and (12) shows that the ratio ε_v of the maximum volumes of the cold and hot cylinder chambers can be used as parameter for optimizing the machine productivity, for the situation when the temperature values, gas mass and nature are identical. If inside the cold machine (at the ambient temperature) the pressure and the gas mass are constant, the total volume occupied by the gas must be constant too (for any value of the ratio ε_v).

The ratio ε_v must be modified keeping constant the total volume V_T occupied by the gas inside the machine. The heats exchanged are maximum for the optimum ratio ε_v. The following condition must be accomplished:

$$\frac{dQ_{ht}}{d\varepsilon_v} = 0, \tag{15}$$

or any other similar derivative of the heats cyclically exchanged in other chambers (because the COP's depend on temperature values only).

The extreme of the function $Q_{ht}(\varepsilon_v)$ is obtained for

$$\varepsilon_{v\,opt} = \sqrt{\tau}, \tag{16}$$

so the optimum volume ratio and the square root of the ratio between the cold and hot source temperatures are equal.

6. Numerical Example

A theoretical VM with the following constructive (fig. 2) and functional characteristics is considered: $d_1 = 0.100$ m, $d_2 = 0.120$ m, $s_1 = s_2 = 0.100$ m, $T_{h1} = 278$ K, $T_k = 343$ K, $T_{h2} = 923$ K. The machine is filled at the environmental temperature $T_0 = 288$ K with hydrogen, until the pressure reaches 5 MPa. After the geometric optimization (with Eq. (16)) the displacer diameters are $d_1 = 0.093$ m and $d_2 = 0.126$ m.

The heats exchanged cyclically inside the optimized machine are $Q_{ht} = -Q_{it2} = 924.7$ J and $Q_{lt} = -Q_{it1} = 2485.1$ J. The coefficient of performance of the VM working as refrigerator is $\varepsilon_r = 2.688$ and as a heat pump is $\varepsilon_{hp} = 3.688$.

The variation of the heat absorbed by the gas inside the high temperature chamber of the theoretical VM, as function of the chamber maximum volume ratio is presented in fig. 4. It can be seen that a relatively broad interval exists for the ratio ε_v inside which the heat exchanged is very close to the maximum one.

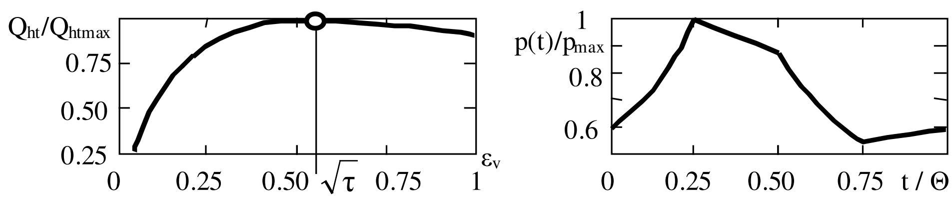

Fig. 4.- Optimum ε_r ratio and pressure variation for theoretical Vuilleumier machine.

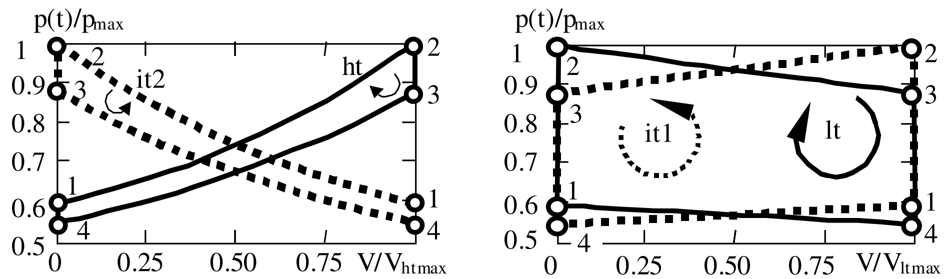

Fig. 5.- Indicator diagrams for theoretical Vuilleumier machine.

The pressure variation inside the VM is nonlinear (fig. 4) and evidentiates the four functional phases that compose the cycle of the theoretical machine.

In fig. 5 the indicator diagrams for all the four VM chambers are presented. The diagrams evidentiate for each chamber the pressure variations at constant volume (maximum or minimum, when its own displacer is halted). All the curves that compose the diagrams represent isothermal processes with variable mass. Each diagram contains one conventional process that closes the cycle inside the chamber, process that takes place with zero mass and at null volume.

7. Conclusions

The paper establishes the requirements that must be accomplished by a Vuilleumier gas thermal machine with two pistons for obtaining the maximum performances. The first requirement that must be accomplished is the correlated movement of the displacers, assuring that each displacer is halted in one of its dead points while the other displacer moves between its own dead points.

It is shown that simultaneous movements of the displacers are not to be desired, because the mass distribution inside the machine chambers in this case becomes incorrect and leads to performance loss. It is shown that a phase angle $\gamma = \pi / 2$ between the movement laws of the displacers assures that the heats exchanged are the maximum ones (for the phase angle γ as variable).

Closed form solutions for theoretical VM performances were deduced. Using the properties of the theoretical VM, the optimum cylinder diameter ratio (for which the heats exchanged are maximum) is deduced.

The definition of the theoretical VM and the analyses of the functional phases duration and of the optimum cylinder diameter ratio are new elements in the study of the Vuilleumier machines. The use of the theoretical VM permits the direct analysis of the influence that some constructive and functional factors have over the machine performances and allows for the emphasizing of some properties that cannot be discerned at the study of the real Vuilleumier machine.

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POMPA DE CĂLDURĂ CU ACŢIONARE TERMICĂ TEORETICĂ
(Rezumat)

Este definită pompa de căldură cu acţionare termică (maşina Vuilleumie)r teoretică, în care gazul evoluează în procese termodinamice simple. Ca rezultat, maşina Vuilleumier teoretică obţine performanţele maxim posibile. Duratele egale ale fazelor funcţionale sînt impuse de condiţia ca mişcarea celor două pistoane să fie defazată cu unghiul γ. Folosind proprietăţile maşinii Vuilleumier teoretice, au fost determinate relaţii exacte pentru performanţele maşinii şi raportul optim al diametrelor cilindrilor pentru care căldurile schimbate sînt maxime.