BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI

Editat de
UNIVERSITATEA TEHNICĂ „GH. ASACHI", IAŞI

Tomul L (LIV)

Fascicula 6C

Secţia
CONSTRUCŢII DE MAŞINI

2004

SECTION 7. THERMOTECHNICS AND THERMAL SYSTEMS

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gh. Asachi", Iaşi,
Tomul L (LIV), Fasc. 6C, 2004
Secţia
CONSTRUCŢII DE MAŞINI

ACME 04619: AN ISOTHERMAL MODEL OF THE VUILLEUMIER MACHINE

Vlad Mario HOMUTESCU*, Emil JUGUREANU*, Condrat Adrian HOMUTESCU*
and Adrian HOMUTESCU **
**Technical University "Gheorghe Asachi" of Iaşi, Horia Street 5-9, Romania,
** S.C. AMADO S.R.L. Iaşi, Romania

Abstract
This paper presents the isothermal model of the Vuilleumier machine. A numerical example with p-V diagrams for the machine chambers is also provided.

Key words: *Vuilleumier machine, isothermal model, heat pump, refrigerator*

1.Introduction

A Vuilleumier machine [1], [2], [3] is a refrigerator (or a heat pump) inside which a constant amount of gas evolves in an almost constant total volume. The gas lies inside several heat exchangers and four variable volume chambers placed (most often) inside two cylinders, each cylinder being fit with its own displacer piston. There are three levels of temperature inside the machine. The refrigerating effect is acquired by expanding the gas inside a low pressure chamber. Pressure variation inside the machine is acquired by heating the agent inside a high temperature chamber and by cooling the agent inside two intermediate temperature chambers.

2.Vuilleumier Machine Construction and Functioning

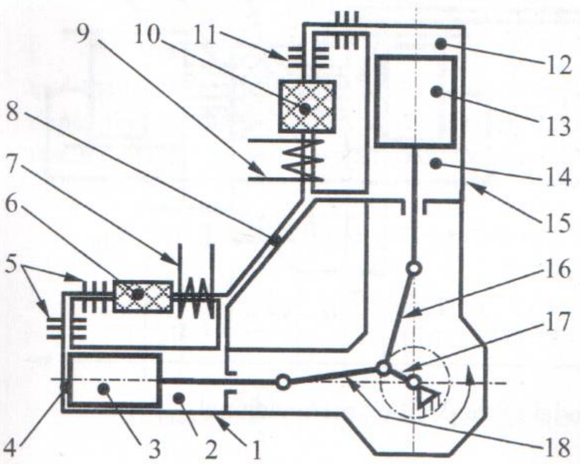

Fig. 1. Vuilleumier machine

According to the schematic diagram in fig. 1 [1], [2], [3], a Vuilleumier machine is comprised of a cold cylinder 1 and a hot cylinder 15 inside which the cold displacer 3 and the hot displacer 13 work. The cold cylinder and displacer share a diameter inferior to the one shared by the hot cylinder and displacer. A drive comprised of crankshaft 17 and rods 16 and 18 provide movement for the displacers. The cold displacer splits the space inside its cylinder in two: a low temperature chamber 4 and an intermediate temperature chamber 2. Inside the hot cylinder the hot displacer delimitates a high temperature chamber 12 and an intermediate temperature chamber 14. Each cylinder is fit with its own heat exchanger set. The cold cylinder has a low temperature heater 5, a low temperature regenerator 6 and an intermediate temperature cooler 7. The hot cylinder is fit with an intermediate temperature cooler 9, a hot temperature regenerator 10 and a high temperature heater 11. The intermediate temperature cooling chambers are connected through pipe 8.

3. Isothermal Physico-mathematical Model for the Vuilleumier Machine

The physico-mathematical model is based on the following hypotheses:

- the working agent is the ideal gas;
- the gas amount evolving inside the machine is constant;
- at thermodynamic level all cycle functional processes are time independent;
- the metallic parts of the machine (other than heat exchangers and cylinder walls) do not exchange heat either among them or with the exterior;
- the processes inside heat regenerators are ideal ones (regeneration efficiencies are 100%); the agent temperature inside the regenerator is deemed constant, being taken as either logarithmic or arithmetic mean;
- the agent temperature inside the low temperature chamber is equal to the one inside the low temperature heater, the one of the outer heating agent, the one of the cylinder walls and the one of the cold displacer frontal surface;
- the agent temperature inside the high temperature chamber is equal to the one inside the high temperature.heater, the one of the outer heating agent, the one of the cylinder walls and the one of the hot displacer frontal surface;
- the agent temperature inside the intermediate temperature chambers is equal to the one inside the coolers, the one of the outer cooling agent, the one of the cylinder walls next to those respective chambers and the one of the stems and of the displacer bottoms;
- the instantaneous pressure is identical in all the spaces occupied by the agent, its value varying along the cycle;
- the volume variation law for the chambers inside the machine cylinders is known.

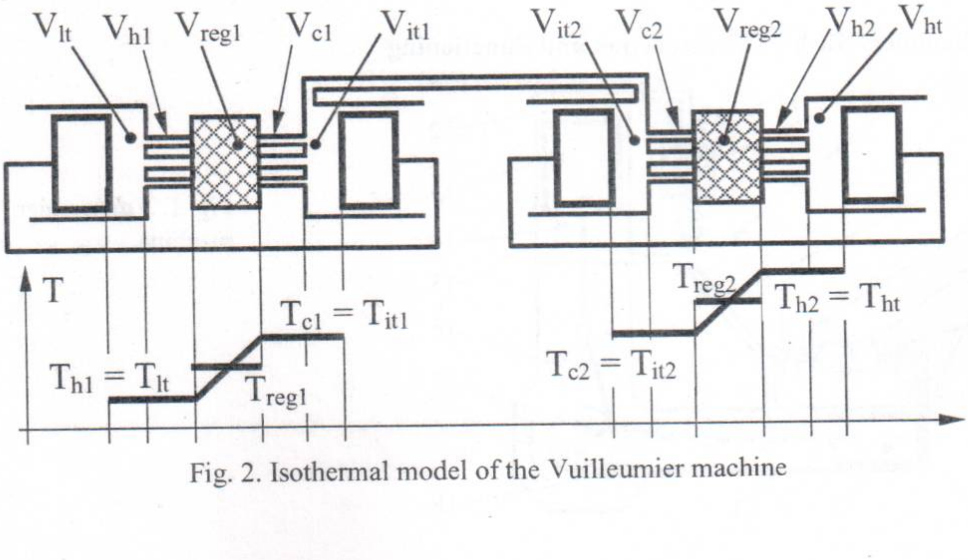

Fig. 2. Isothermal model of the Vuilleumier machine

The hypotheses implying the temperatures inside the Vuilleumier machine show that inside all chambers isothermal processes take place only, thus confirming the described physico-mathematical model the denomination of isothermal model. To outline the isothermal character of the physico-mathematical model analyzed here, on fig. 2 the machine chambers are separate and placed in row. This presentation required the halving of each displacer. Each variable volume chamber is assigned half a displacer. The mechanical linkage between the displacer halves was symbolically drawn through bars exterior to the cylinder.

We used the following subscripts for dimensions inside machine chambers (volume V, temperature T, mass m): h = heater; reg = regenerator; c = cooler; 1 = cold displacer; 2 - hot displacer; ht = high temperature; lt = low temperature; it = intermediate temperature.

The pressure variation law $p(\alpha)$ was determined from the equation of conservation for the total gas mass inside the machine:

$$m = \sum_i m_i , \quad (1)$$

where the masses m_i are calculated from the equation of state for each chamber:

$$m_i = \frac{p(\alpha)\, V_i(\alpha)}{R\, T_i} . \quad (2)$$

The pressure acquires the expression:

$$p(\alpha) = \frac{m\, R}{\sum_i \frac{V_i(\alpha)}{T_i}} \quad (3)$$

where

$$\sum_i \frac{V_i(\alpha)}{T_i} = \frac{V_{lt}(\alpha) + V_{h1}}{T_{lt}} + \frac{V_{reg1}}{T_{reg1}} + \frac{V_{c1} + V_{it1}(\alpha) + V_{it2}(\alpha) + V_{c2}}{T_c} + \frac{V_{reg2}}{T_{reg2}} + \frac{V_{h2} + V_{ht}(\alpha)}{T_{ht}} . \quad (4)$$

Temperature values T_{lt}, T_c, T_{ht} are imposed and the chamber volumes are imposed by the geometry chosen for the machine. Regenerator mean temperatures are calculated with relations:

$$T_{reg1} = \frac{T_c - T_{lt}}{\ln(T_c / T_{lt})} \text{ and } T_{reg2} = \frac{T_{ht} - T_c}{\ln(T_{ht} / T_c)} \quad (5)$$

and the mathematical expressions of the chamber volume variation laws inside the cylinders (laws depending on the drive type and the coordinate system chosen) are calculated, for the Vuilleumier machine in fig. 1, with respect to the schematic diagram in fig. 3:

$$V_{ht}(\alpha) = \frac{\pi D_2^2}{4}\left[f_{TDP2} + l_2 + r - r\cos(\alpha) - l_2\cos(\beta_2(\alpha))\right] \quad (6)$$

$$V_{it2}(\alpha) = \frac{\pi\left(D_2^2 - d_2^2\right)}{4}\left[f_{BDP2} - (l_2 - r) + r\cos(\alpha) + l_2\cos(\beta_2(\alpha))\right] \quad (7)$$

$$V_{lt}(\alpha) = \frac{\pi D_1^2}{4}\left[f_{TDP1} + l_1 + r - r\cos(\alpha - \gamma) - l_1\cos(\beta_2(\alpha - \gamma))\right] \quad (8)$$

$$V_{it1}(\alpha) = \frac{\pi\left(D_1^2 - d_1^2\right)}{4}\left[f_{BDP1} - (l_1 - r) + r\cos(\alpha - \gamma) + l_1\cos(\beta_2(\alpha - \gamma))\right]. \quad (9)$$

The following dimensions were used: D = displacer diameter; d = displacer stem diameter; f_{TDP} = distance between the displacer at the top dead point and the cylinder head; f_{BDP} = distance between the displacer at the bottom dead point and the cylinder head; l = rod length; r = crank length; β = rod position angle; γ = angle between cylinder axes.

For the variation laws of the chamber volumes inside the cold cylinder it was taken into account that the drive behaves similarly to the warm displacer yet delayed with angle γ.

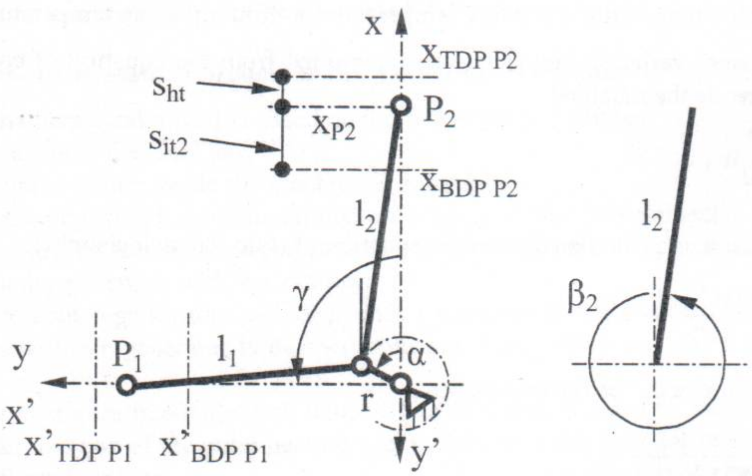

Fig. 3. Schematic diagram for determining the variation laws of the chamber volumes inside Vuilleumier machine chambers cylinders

Accordingly to the first law of thermodynamics applied to the agent undergoing a cycle inside an isothermal chamber, the heat exchanged is equal to the work exchanged and is calculated with the relations:

$$Q_{lt} = \int_0^{2\pi} p(\alpha)\left[\frac{dV_{lt}(\alpha)}{d\alpha}\right]d\alpha; \qquad Q_{it1} = \int_0^{2\pi} p(\alpha)\left[\frac{dV_{it1}(\alpha)}{d\alpha}\right]d\alpha; \quad (10)$$

$$Q_{it2} = \int_0^{2\pi} p(\alpha)\left[\frac{dV_{it2}(\alpha)}{d\alpha}\right]d\alpha; \qquad Q_{ht} = \int_0^{2\pi} p(\alpha)\left[\frac{dV_{ht}(\alpha)}{d\alpha}\right]d\alpha. \quad (11)$$

As a consequence of the displacer stem's presence, the machine exchanges with the environment an amount of work per cycle having the expression

$$L = Q_{lt} + Q_{it1} + Q_{it2} + Q_{ht}. \quad (12)$$

The useful effect of the Vuilleumier refrigerating machine is represented by heat Q_{lt} extracted from the low temperature heat source and the useful effect of the heat pump is heat (Q_{lt} +Q_{ht}) transmitted to the user at an intermediate temperature of T_c.

The Vuilleumier refrigerating machine is characterized by the coefficient of performance (heat rejected/heat consumed)

$$\varepsilon_r = \frac{|Q_{lt}|}{Q_{ht}} \qquad (13)$$

and the heat pump by the coefficient of performance

$$\varepsilon_{hp} = \frac{|Q_{it1} + Q_{it2}|}{Q_{ht}}. \qquad (14)$$

The pressure increase ratio inside the machine is

$$\pi = \frac{p_{max}}{p_{min}}. \qquad (15)$$

4.Numerical example

Vuilleumier machine featuring the following dimensions is assumed: D_1= 0,073 m; d_1= d_2 = 0,01 m; D_2 = 0,063 m; r_1 = r_2 = 0,0365 m; l_1= l_2 = 0,15 m; f_{TDP1} = f_{BDP1} = f_{TDP2} = f_{BDP2} = 0,001 m; V_{h1} = V_{h2} = V_{c1} = V_{c2} = 0,05 V_{Sd1}; V_{reg1} = V_{reg2} = 1,2 V_{Sd2}, where V_{Sd2} = 0,3055 10^{-3} m^3 (volume swept by the high temperature displacer).

The machine works with a total mass of hydrogen m = 0,004 kg (R = 4121 J/(kg K) between temperatures T_{ht} = T_{h1} = 773 K; T_{c1} = T_{c2} = T_{it1} = T_{it2} = 313 K and T_{lt} = T_{h2} = 273 K.

The numerical solving of the described Vuilleumier machine isothermal model equations lead to the results displayed in fig. 4 and fig. 5, as well as inside table 1.

Table 1

Q_{ht}	Q_{lt}	Q_{it1}	Q_{it2}	L	ε_r	ε_{hp}	p_{max}/p_{min}
[J/cycle]					-	-	-
37,7	126,5	-123,3	-37,0	· 3,9	3,356	4,253	1,172

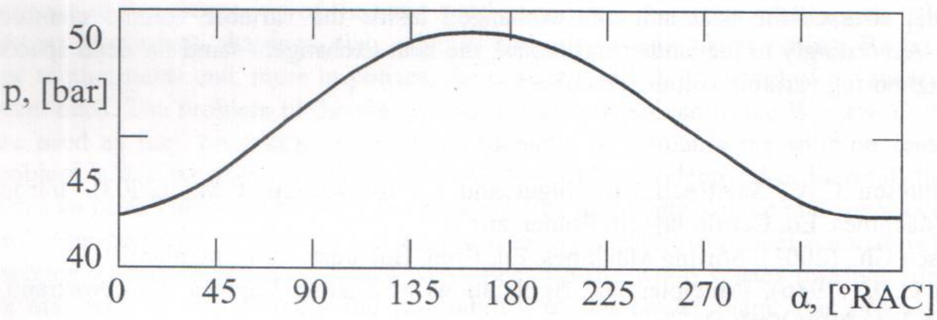

Fig. 4. Pressure variation inside Vuilleumier machine

5.Conclusions

The physico-mathematical model proposed for the numerical simulation of the Vuilleumier machine functioning allows for providing information on the possible maximum performance the machine is capable of. Inside a real machine the heat exchanges do not take place isothermal, the heat regeneration is not ideal and the agent flow through the heat exchangers occurs with friction, all these facts lowering the performance beneath the isothermal one.

The isothermal model allows for a rapid analysis of the influence some constructive and functional factors have as well a s for comparing different machines.

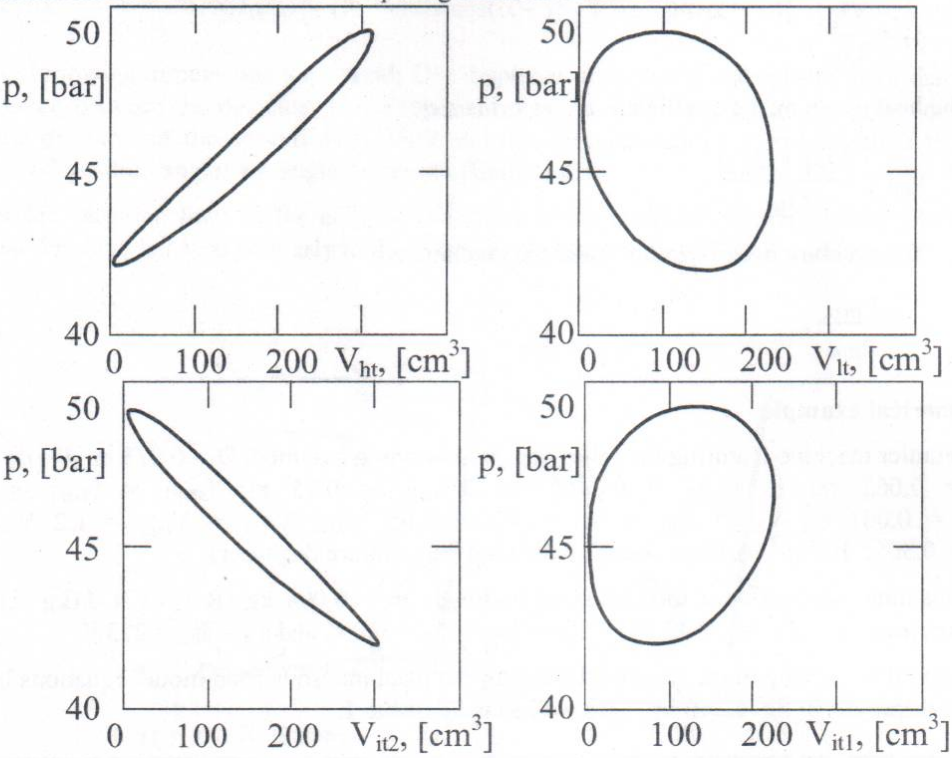

Fig. 5. Pressure-volume diagrams for Vuilleumier machine chambers

The energetic balance per cycle for the Vuilleumier machine, written in expression (14), shows that, because of the piston stems, the machine also produces a small amount of work which, inside the real machine, is insufficient to compensate for the friction losses.

The model stresses the heat amounts exchanged inside the variable volume chambers of the machine. Accordingly to the isothermal model, the heat exchangers stand for dead spaces attached to the neighboring variable volume chambers.

References

[1] Homutescu C.A., Savitescu Gh., Jugureanu E., Homutescu V.M., (2003), Introduction in Stirling Machines. Ed. Cermi, Iaşi (in Romanian).
[2] Popescu Gh., (2001), Stirling Machines. Ed. Bren, Bucureşti (in Romanian).
[3] West C.D., (1986), Principles and Applications of Stirling Engines. Van Nostrand Reinhold Company, Inc., New York, 247 p.

MODEL IZOTERMIC AL MAŞINII VUILLLEUMIER

(Rezumat)

Este prezentat un model fizico-matematic izotermic al maşinii Vuilleumier. Sunt date relaţiile de calcul a variaţiei presiunii şi a performanţelor teoretice izotermice ale maşinii. Un exemplu de calcul ilustrează aplicarea modelului.