THERMAL AND ELECTRICAL POWER GENERATION

5TH INTERNATIONAL CONFERENCE ON ELECTROMECHANICAL AND POWER SYSTEMS
October 6-8, 2005 - Chisinau, Rep.Moldova

SEMI-ADIABATIC PHYSICO-MATHEMATICAL MODEL OF THE VUILLEUMIER HEAT PUMP

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Abstract – *The paper presents a semi-adiabatic model of the Vuilleumier machine. The model assumes the hypothesis that inside the low temperature and inside the high temperature chambers adiabatic processes take place only. Inside the intermediate temperature chambers and inside the heat exchangers only isothermal processes take place. The differential equations of the processes inside the heat pump are deduced.*

Keywords – *Vuilleumier machine, semi-adiabatic model, thermal-acted heat pump.*

1. INTRODUCTION

A thermal-acted heat pump (a Vuilleumier machine) [1], [2], [3], [4] is a machine inside which a constant amount of gas evolves inside an almost constant total volume. The gas lies inside several heat exchangers and four variable volume chambers placed (most often) inside two cylinders, each cylinder being fit with its own displacer piston. There are three levels of temperature inside the machine. The refrigerating effect is acquired by expanding the gas inside a low temperature chamber. Pressure variation inside the machine is acquired by heating the agent inside a high temperature chamber and by cooling the agent inside two intermediate temperature chambers.

2. VUILLEUMIER MACHINE CONSTRUCTION AND FUNCTIONING

According to the schematic diagram in fig. 1 [1], [2], [3], a Vuilleumier machine is comprised of a cold cylinder 1 and a hot cylinder 15 inside which the cold displacer 3 and the hot displacer 13 work. The cold cylinder and displacer share a diameter inferior to the one shared by the hot cylinder and displacer. A drive comprised of crankshaft 17 and rods 16 and 18 provide movement for the displacers. The cold displacer splits the space inside its cylinder in two chambers: a low temperature one 4 and an intermediate temperature one 2. Inside the hot cylinder the hot displacer delimitates a high temperature chamber 12 and an intermediate temperature chamber 14. Each cylinder is fit with its own heat exchanger set. The cold cylinder has a low temperature heater 5, a low temperature regenerator 6 and an intermediate temperature cooler 7. The hot cylinder is fit with an intermediate temperature cooler 9, a hot temperature regenerator 10 and a high temperature heater 11. The intermediate temperature cooling chambers are connected through pipe 8.

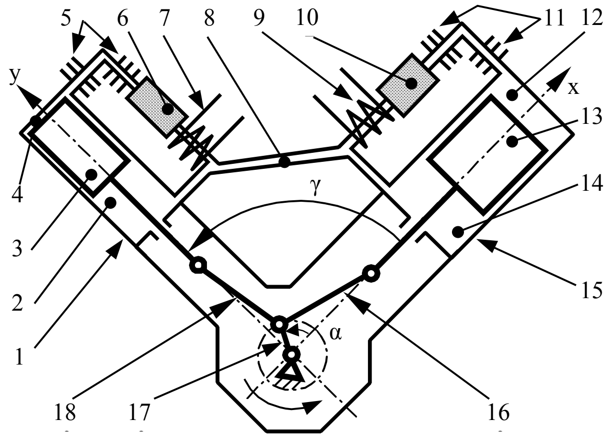

Fig. 1 - Vuilleumier machine

3. THE HYPOTHESES OF THE MODEL

The semi-adiabatic model is based on the following hypotheses:

- the working agent is the ideal gas,
- the gas amount inside the machine is constant,
- at thermodynamic level all cycle functional processes are time independent,
- the metallic parts of the machine (other than the heaters and the cylinder walls confining the intermediate temperature chambers) do not exchange heat either among them or with the exterior,
- the processes inside heat regenerators are ideal ones (regeneration efficiencies are 100%); the agent temperature inside the regenerator is deemed constant, being taken as arithmetic or logarithmic mean,
- inside the low temperature chamber 4 (on fig. 1) and inside the high temperature chamber 12 adiabatic processes take place; so, the temperature inside these chambers vary cyclically,
- the agent temperature inside the intermediate temperature chambers is equal to the one of the coolers, to the one of the outer cooling agent, the one of the cylinder walls next to their respective chambers and the one of the stems and of the displacer bottoms,
- inside the coolers and heaters only isothermal processes take place,
- the instantaneous pressure is identical in all the spaces occupied by the agent, its value varying along the cycle,
- the movement of the displacers is the real movement, given by the crankshaft.

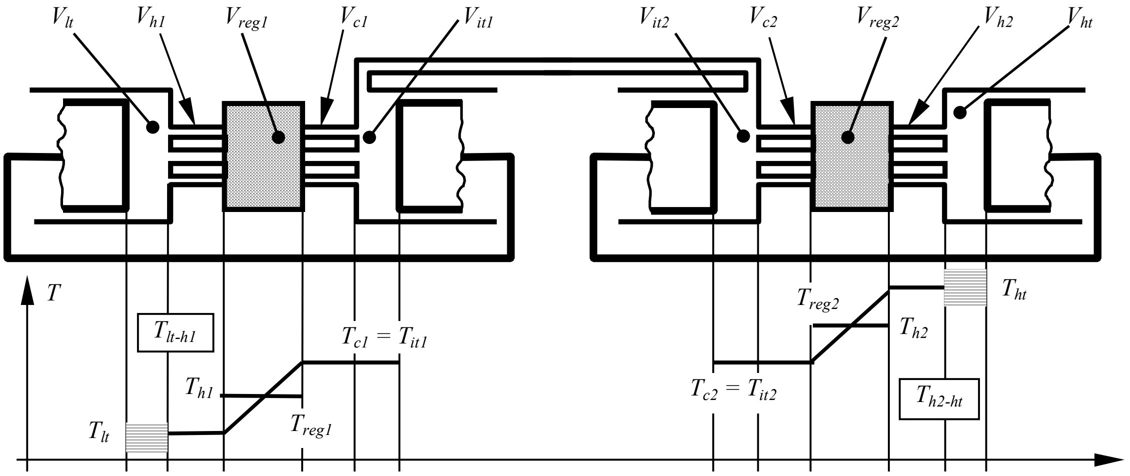

Fig. 2 - Semi-adiabatic model of the Vuilleumier machine

The hypotheses implying the temperatures inside the Vuilleumier machine show that inside two of the machine chambers take place adiabatic processes and inside all other chambers isothermal processes take place only, thus confirming the described physico-mathematical model the denomination of semi-adiabatic model (this denomination is used also by West [4]). To outline the semi-adiabatic character of the physico-mathematical model analyzed here, on fig. 2 the machine chambers are separate and placed in row. This presentation required the halving of each displacer. Each variable volume chamber is assigned half a displacer. The mechanical linkage between the displacer halves was symbolically drawn through bars exterior to the cylinder.

The following subscripts for dimensions inside machine chambers (volume *V*, temperature *T*, mass *m*) were used: *h* = heater; *reg* = regenerator; *c* = cooler; *1* = cold displacer; *2* - hot displacer; *ht* = high temperature; *lt* = low temperature; *it* = intermediate temperature; *T* = total. The composed subscripts *lt-h1* and *h2-ht* refer to the dimensions describing the separating sections between the low and high temperature chambers and its adjacent heaters.

4. THE SEMI-ADIABATIC PHISICO-MATHEMATICAL MODEL

The model uses the differential equation of the conservation of the working agent total mass, the equation of state applied to the heat exchangers and to the intermediate temperature chambers and the differential law of conservation of energy written for the adiabatic chambers [5].

The differential equation of the conservation of the working agent total mass is

$$\begin{aligned} d(m_T) = {} & d(m_{lt}) + d(m_{h1}) + d(m_{reg1}) + d(m_{c1}) + \\ & + d(m_{it1}) + d(m_{it2}) + d(m_{c2}) + d(m_{reg2}) \\ & + d(m_{h2}) + d(m_{ht}) = 0\,. \end{aligned} \tag{1}$$

The differential expressions of the agent masses inside the heat exchangers are obtained from the equation of state, in which *V* = const. and *T* = const.:

$$\frac{dp}{p} = \frac{dm}{m}\,. \tag{2}$$

The mass *m* is taken from the equation of state and the differential expression of the mass inside a generic heat exchanger becomes:

$$dm_j = \frac{V_j}{RT_j}dp\,. \tag{3}$$

where the subscript *j* is replaced by *h1*, *reg1*, *c1*, *c2*, *reg2* and *h2*.

For the intermediate temperature chambers (2 and 14 on fig. 1), the equation of state written at *T* = const. in differential form becomes:

$$\frac{dp}{p} + \frac{dV}{V} = \frac{dm}{m}\,. \tag{4}$$

For the mass differential expression the following form is obtained:

$$dm_i = \frac{1}{RT_i}(p\,dV_i + V_i\,dp)\,, \tag{5}$$

where *i* is replaced by *it1* and *it2*.

Accordingly to the adopted hypotheses, inside the low temperature chamber (4, on fig. 1) and inside the high temperature chamber (12, on fig. 1) the gas exchanges work with the surrounding environment (through piston movement) and enthalpy with the neighbouring chambers (through the agent's entering the chamber from the neighbouring heater or leaving it toward the heater). The internal energy of the gas inside the adiabatic chamber changes, as a consequence of mass and temperature variations. Inside these two chambers the heat exchanged is zero, conforming to the adiabatic hypothesis. The energy balance takes the expression:

$$dL + dU + dI = 0\,. \tag{6}$$

For the low temperature chamber the terms in (6) are explicated by the following relations:

$$dL_{lt} = p\,dV_{lt} \ , \tag{7}$$

$$dU_{lt} = d(c_v\, m_{lt}\, T_{lt}) = \frac{c_v}{R}(V_{lt}\, dp + p\, dV_{lt}) \ , \tag{8}$$

$$dI_{lt-h1} = d(c_p\, m_{lt-h1}\, T_{lt-h1}) = -c_p\, T_{lt-h1}\, dm_{lt} \ . \tag{9}$$

Equation (9) takes into account that $dm_{lt-h1} = -dm_{lt}$, because the mass of working agent that passes through the section *lt-h1* is equal to the variation of the mass of the gas inside the chamber, taken with opposite sign. The positive sense of the agent flow inside the machine is considered to be from the low temperature chamber toward the high temperature chamber. The term $c_p\, m_{lt}\, dT_{lt-h1}$ was neglected, assuming the hypothesis that it is small in comparison with the other term.

Introducing (7), (8) and (9) in (6) and explaining the mass differential, the following relation is obtained:

$$dm_{lt} = \frac{1}{R\, T_{lt-h1}} \left[p\, dV_{lt} + \frac{V_{lt}}{k} dp \right]. \tag{10}$$

Similarly, for the high temperature chamber the next expression is obtained

$$dm_{ht} = \frac{1}{R\, T_{h2-ht}} \left[p\, dV_{ht} + \frac{V_{ht}}{k} dp \right]. \tag{11}$$

Introducing the expressions of the mass differentials for the ten chambers of the machine, given by (3), (5), (10) and (11) in (1), after some algebraic operations, the differential expression of the pressure is obtained:

$$dp = A / B \, , \tag{12}$$

where the terms A and B are

$$A = -k\, p \left[\frac{dV_{lt}}{T_{lt-h1}} + \frac{dV_{ht}}{T_{h2-ht}} + \frac{dV_{it1}}{T_{it1}} + \frac{dV_{it2}}{T_{it2}} \right] ; \tag{13}$$

$$\begin{aligned} B = {} & \frac{V_{lt}}{T_{lt-h1}} + \frac{V_{ht}}{T_{h2-ht}} + k \left(\frac{V_{it1}}{T_{it1}} + \frac{V_{it2}}{T_{it2}} \right) + \\ & + k \left(\frac{V_{h1}}{T_{h1}} + \frac{V_{reg1}}{T_{reg1}} + \frac{V_{c1}}{T_{c1}} + \frac{V_{c2}}{T_{hc2}} + \frac{V_{reg2}}{T_{reg2}} + \frac{V_{h2}}{T_{h2}} \right). \end{aligned} \tag{14}$$

For the low and high temperature chambers the differential expression of the temperature is taken from the equation of state:

$$\frac{dp}{p} + \frac{dV}{V} = \frac{dm}{m} + \frac{dT}{T} \ . \tag{15}$$

Particularizing for the two adiabatic chambers, the following relations are obtained

$$dT_{lt} = T_{lt} \left(\frac{dp}{p} + \frac{dV_{lt}}{V_{lt}} - \frac{dm_{lt}}{m_{lt}} \right), \tag{16}$$

$$dT_{ht} = T_{ht} \left(\frac{dp}{p} + \frac{dV_{ht}}{V_{ht}} - \frac{dm_{ht}}{m_{ht}} \right). \tag{17}$$

Equations (12), (10), (11), (16) and (17) form the system of differential equations of the semi-adiabatic physico-mathematical model of the Vuilleumier machine. The unknown functions are the pressure p, the masses m_{lt} and m_{ht} inside the low and high temperature chambers and the temperatures T_{lt} and T_{ht} in the same chambers. The system is non-linear, because there are several terms in the differential equations that have an order higher than one. The system has variable coefficients and the conditional temperatures T_{lt-h1} and T_{h2-ht} of the agent that passes through the surfaces *lt-h1* and *h2-ht* depend on the sense of the gas flow. The conditional temperatures take the expressions:

$$\begin{aligned} & T_{lt-h1} = T_{lt} \quad \text{if} \quad m_{lt-h1} > 0 \quad (\text{or} \quad dm_{lt} < 0); \\ & T_{lt-h1} = T_{h1} \quad \text{if} \quad m_{lt-h1} < 0 \quad (\text{or} \quad dm_{lt} > 0); \\ & T_{h2-ht} = T_{h2} \quad \text{if} \quad m_{h2-ht} > 0 \quad (\text{or} \quad dm_{ht} > 0); \\ & T_{h2-ht} = T_{ht} \quad \text{if} \quad m_{h2-ht} < 0 \quad (\text{or} \quad dm_{ht} < 0). \end{aligned} \tag{18}$$

The system can be solved only by numerical integration. If the values of the unknown functions are adopted for certain point in time, the problem is an initial value one and the numerical solution can be found with a Runge-Kutta method. The solution is obtained after several iterations, each of them using the previous one's results as initial values and thus getting closer to the result as the analysis goes on.

As a consequence of the displacer stem's presence, the machine exchanges with the environment an amount of work per cycle having the expression

$$L = Q_{h1} + Q_{it1} + Q_{it2} + Q_{h2} \, . \tag{19}$$

The heat amounts exchanged in the machine are calculated from the energy balance.

The coefficient of performance of the heat pump is

$$\varepsilon_{hp} = \frac{\left| Q_{it1} + Q_{it2} \right|}{Q_{ht}} \, . \tag{20}$$

5. NUMERICAL EXAMPLE

A Vuilleumier machine featuring the following dimensions is assumed: D_1= 0,1 m; d_1= d_2 = 0,02 m; D_2 = 0,12 m; $r_1 = r_2$ = 0,05 m; l_1= l_2 = 0,2 m; $f_{TDP1} = f_{BDP1} = f_{TDP2} = f_{BDP2}$ = 0,001 m; $V_{h1} = V_{h2} = V_{c1} = V_{c2}$ = 0,05 V_{Sd1}; $V_{reg1} = V_{reg2}$ = 1,2 V_{Sd2}, where V_{Sd2} = volume swept by the high temperature displacer, D = cylinder diameter, d = stem diameter, r = crankshaft radius, l = rod length, f = dead space length.

The machine works with a total mass of hydrogen m = 0.0207 kg (corresponding to a pressure of 50 bar in the machine, at an ambient temperature of 15 ºC; R_{H2} = 4121 J/(kg K)) between temperatures $T_{ht} = T_{h2}$ = 923 K;

$T_{c1} = T_{c2} = T_{it1} = T_{it2} = 343$ K and $T_{lt} = T_{h1} = 278$ K.
The numerical solving of the described Vuilleumier machine semi-adiabatic model equations lead to the results displayed in fig. 3, fig. 4 and fig. 5, as well as inside table 1.

Table 1 - Calculated results

Model	Q_{lt} / Q_{h1}	Q_{ht} / Q_{h2}	Q_{it1}	Q_{it2}	ε_{hp}
	[J/cycle]				-
isothermal	718.0	300.4	-700.9	-295.6	3.32
semi-adiabatic	749.4	350.3	-732.4	-344.7	3.07

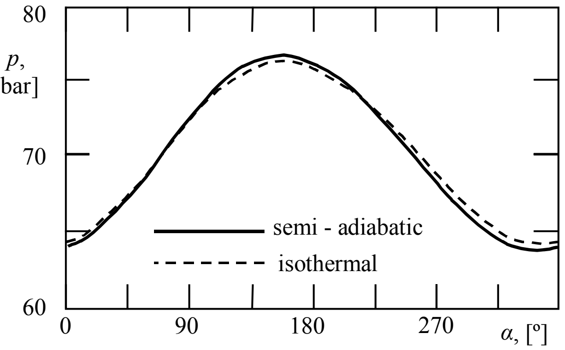

Fig. 3 - Pressure variation inside Vuilleumier machine

The pressure variation inside the machine is shown in fig. 3 and the temperature variations in the adiabatic chambers are shown in fig. 4 and fig. 5.

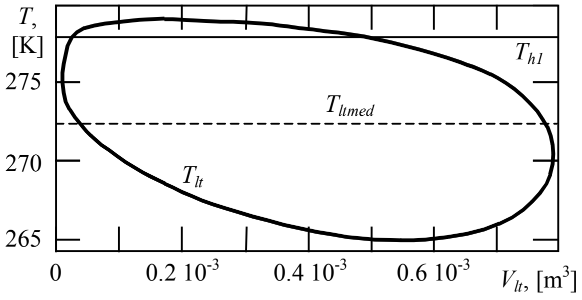

Fig. 4 - Temperature variation inside the low temperature chamber

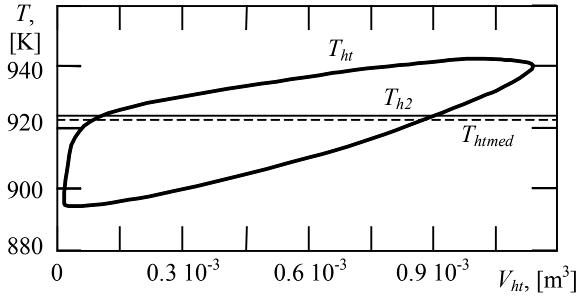

Fig. 5 - Temperature variation inside the high temperature chamber

6. CONCLUSIONS

The physico-mathematical semi-adiabatic model proposed for the numerical simulation of the Vuilleumier machine functioning allows for providing information on the possible performance the machine is capable of. Inside a real machine the heat exchanges do not take place isothermally, the heat regeneration is not ideal and the agent flow through the heat exchangers occurs with friction, all these facts lowering the performance beneath the semi-adiabatic one.

The pressure variation inside the machine, calculated with the semi-adiabatic model, is very close to the one calculated with the isothermal model [2], because a large quantity of the working agent is placed inside chambers considered to be isothermal in both models.

In the semi-adiabatic model the temperature inside the low temperature chamber is - for the most part of the cycle - below the neighbouring isothermal heater temperature. The mean temperature T_{ltmed} inside this adiabatic chamber is below the cold heater temperature.

The model stresses the heat amounts exchanged inside the machine chambers. Accordingly to the adopted hypotheses, the coolers stand for dead spaces attached to the neighbouring variable volume chambers. Because the heaters are adjacent to the adiabatic chambers, they cyclically exchange nonzero heats.

The heat amounts exchanged with the heat sources using the semi-adiabatic model are larger than the corresponding ones calculated with the isothermal model. The coefficient of performance (COP) is smaller than the isothermal one.

The semi-adiabatic model allows for a rapid analysis of the influence some constructive and functional factors have (more than the isothermal model can support) as well as for comparing different machines.

The energetic balance per cycle for the Vuilleumier machine, written in expression (19), shows that the machine produces a small amount of work also, as result of the piston stem presence. Inside the real machine, this amount is insufficient to compensate for the friction losses.

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