SOCIETATEA ROMÂNĂ A TERMOTEHNICIENILOR
UNIVERSITATEA PETROL-GAZE DIN PLOIEŞTI

CONFERINŢA NAŢIONALĂ DE TERMOTEHNICĂ CU PARTICIPARE INTERNAŢIONALĂ

EDIŢIA a XVI -a
31 mai - 1 iunie 2007
PLOIEŞTI

Volumul 1
TERMODINAMICĂ, SCHIMB DE CĂLDURĂ ŞI APLICAŢII

EDITURA UNIVERSITĂŢII PETROL - GAZE DIN PLOIEŞTI

The Impossibility of Modeling the Thermodynamic Stirling Cycle on Gamma-Type Machines

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Abstract

The paper analyzes the gamma-type theoretical Stirling machine. It was showed that on such a machine the thermodynamic Stirling cycle could not be obtained. The gamma-type Stirling motor functioning and the energy amounts exchanged with the exterior were analyzed. It was showed that the efficiency of the gamma-type isothermal motor with heat regeneration is equal to the efficiency of a Carnot cycle working between the same extreme temperatures, due to the isothermal hypotheses. The work produced is less than the one of the other Stirling engines (for the same conditions - volumes, temperatures, gas mass).

Key words: theoretical Stirling machine, gamma type motor, p-V diagram, performance

Introduction

In the classical theory of the Stirling machines [3], [4] the thermodynamic cycle with the same name (fig. 1a) can be achieved in theoretical machines with two pistons. The thermodynamic Stirling cycle is composed from two isothermal processes and two isochoric processes that take place at the temperatures of the heat sources (T_c for the compression process and T_e for the expansion process) and at the extreme volumes V_m and V_M [1]. The scheme of the Stirling machine is presented in fig. 1b. The model machine has two pistons that delimit the compression and the expansion chambers. The machine is fitted with a heat regenerator (with 100 % efficiency) and with a heater and a cooler.

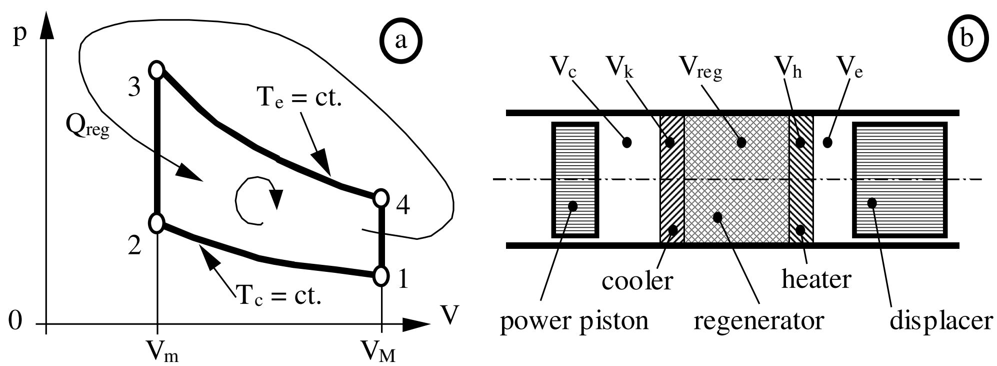

Fig. 1. The Stirling cycle with heat regeneration (a) and the scheme of the Stirling engine with heat exchangers that contain void spaces (b)

In alpha and beta-type Stirling machines (fig. 2) the gas can evolve in a Stirling thermodynamic cycle and the functioning can be modeled using a set of hypotheses [2]. The most important hypotheses imposed isothermal evolutions of the gas inside the machine chambers.

Stirling Theoretical Machine

The theoretical Stirling machine is a machine in which the working agent performs the thermodynamic Stirling cycle [2].

The compression isotherm 1-2 takes place at the cycle minimum temperature, so that it is impossible to have gas amounts with higher temperatures inside other chambers. As a consequence, the volumes of the regenerator and of the heater must be zero. During the compression the expansion chamber must also have null volume, so the displacer must halt near the cylinder head. The expansion isotherm 3-4 takes place at the cycle maximum temperature, so that it is impossible to have gas amounts with lower temperatures inside other chambers. As a consequence, the volume of the cooler is null, and during the expansion the volume of the compression chamber must be null. From the two previously enunciated conditions it results that the theoretical Stirling engine must have only two chambers, one for compression and one for expansion. The realization of the isochoric process 4-1 imposes that the maximum volume of the expansion chamber must be equal to the maximum volume of the compression chamber. The realization of the isochoric process 2-3 imposes that the minimum volume of the expansion chamber must be equal to the minimum volume of the compression chamber.

The machine chambers must maintain their functional role over the entire period of the cycle. As a consequence, inside the compression and expansion chambers processes at constant temperature can take place only.

Inside the theoretical Stirling machine the heat transfer is realized through portions of the cylinder and of the cylinder head. The regenerator is represented also by a portion of the cylinder walls. Inside the theoretical Stirling machine a constant mass of ideal gas evolves. The pressure losses cannot occur and the instantaneous pressure is the same inside all machine chambers.

The duration of the functional phases are arbitrary, because at the theoretical machine the performances are given by the law of variation of pressure as function of volume and not of time. The piston movement laws are also arbitrary, for the same reason: the performances depend only on the p(V) variation law. The periods during which the pistons remain stationary and the periods during which the pistons have correlated movements depend on the constructive scheme of the machine.

In conclusion, inside the theoretical Stirling machine a constant mass of working gas evolves inside two isothermal chambers. The chamber volumes vary between zero and identical maximum values. Depending on the way of placing the pistons in one or in two cylinders, the classic theory defines three constructive schemes for Stirling machines: alpha, beta and gamma [1], [3]. The theoretical Stirling machines (in which the gas evolves in a Stirling cycle, because all the previous stated conditions are satisfied) can be realized only on the alpha and beta schemes (fig. 2 α and β) [2].

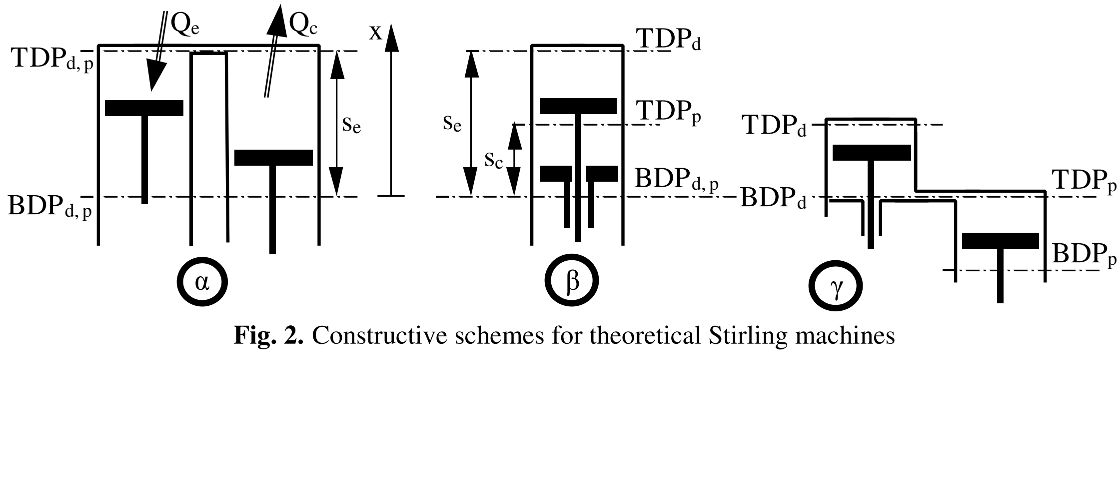

Fig. 2. Constructive schemes for theoretical Stirling machines

Gamma-Type Theoretical Machine

Because during the expansion process a fraction of the total mass of gas lies inside the compression chamber [2], inside the gamma-type machine the gas cannot evolve in a Stirling thermodynamic cycle. For a correct comparison of the gamma machine (fig. 2 γ) with the alpha and beta-type Stirling machines, in all these machines the same mass of gas must evolve and the maximum volume of the compression chamber $V_{max\ c}$ must be the same. The temperatures of the heat sources are the same. For a theoretical gamma-type machine the condition $V_{max\ c}$ = const. requires equal piston strokes (for the same cylinder diameters). On fig. 2, the strokes are half of the beta or alpha displacer stroke, in order to obtain the same (chosen) compression ratio. The power piston stands still during the gas transfer phases and the displacer stands still during the compression and the expansion phases.

In the classical theory of the Stirling machines [3], [4] the compression chamber is treated as a single functional unit, in spite of the fact that it is composed from two distinct variable spaces. At the gamma-type motor the variable volumes inside the displacer cylinder do not contribute to the work exchanged with the exterior, because the pressure difference between the two sides of the displacer is zero. So, the two compression spaces play different functional roles and must be analyzed separately. The scheme of a theoretical gamma-type motor is presented in fig. 3. This presentation required the halving of the displacer. To each variable volume chamber inside the displacer cylinder half a displacer is assigned. The mechanical linkage between the displacer halves was symbolically drawn through a bar outside the cylinder.

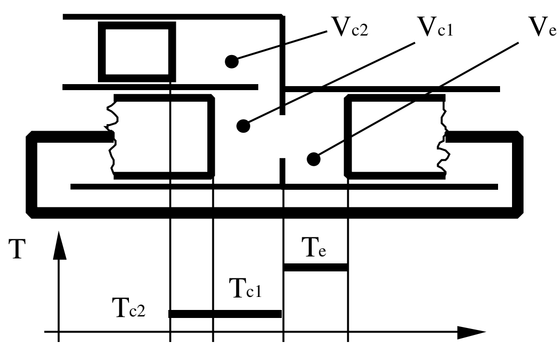

Fig. 3. Scheme of a theoretical gamma-type machine

The variation of the instantaneous pressure inside the theoretical machine is obtained from the equation of the mass conservation inside the machine and has the expression

$$p = \frac{m_T R}{\frac{V_c}{T_c} + \frac{V_e}{T_e}} = \frac{m_T R}{\frac{V_{c1}}{T_c} + \frac{V_{c2}}{T_c} + \frac{V_e}{T_e}}, \quad (1)$$

where R is the individual gas constant and the subscript T means total.

The analysis of the functioning, described by the curves on fig. 4, shows that inside the gamma-type machine only one process with constant mass takes place, i.e. the compression 1-2. The expansion 3-4 is a process with variable mass, the gas lying partially inside the expansion chamber and partially inside the compression chamber. The expansion process for the whole machine 3-4 is no longer isothermal and so inside the gamma-type machine the gas cannot evolve in a Stirling cycle (composed from two isotherms connected by two isochores).

Inside the compression chamber the following processes take place: the isothermal compression (with constant mass m_T - the process 1_c-2_c on fig. 3d), the isothermal processes of gas exchange with the expansion chamber (with variable mass, 2_c-3_c, 3_c-4_c and 4_c-1_c).

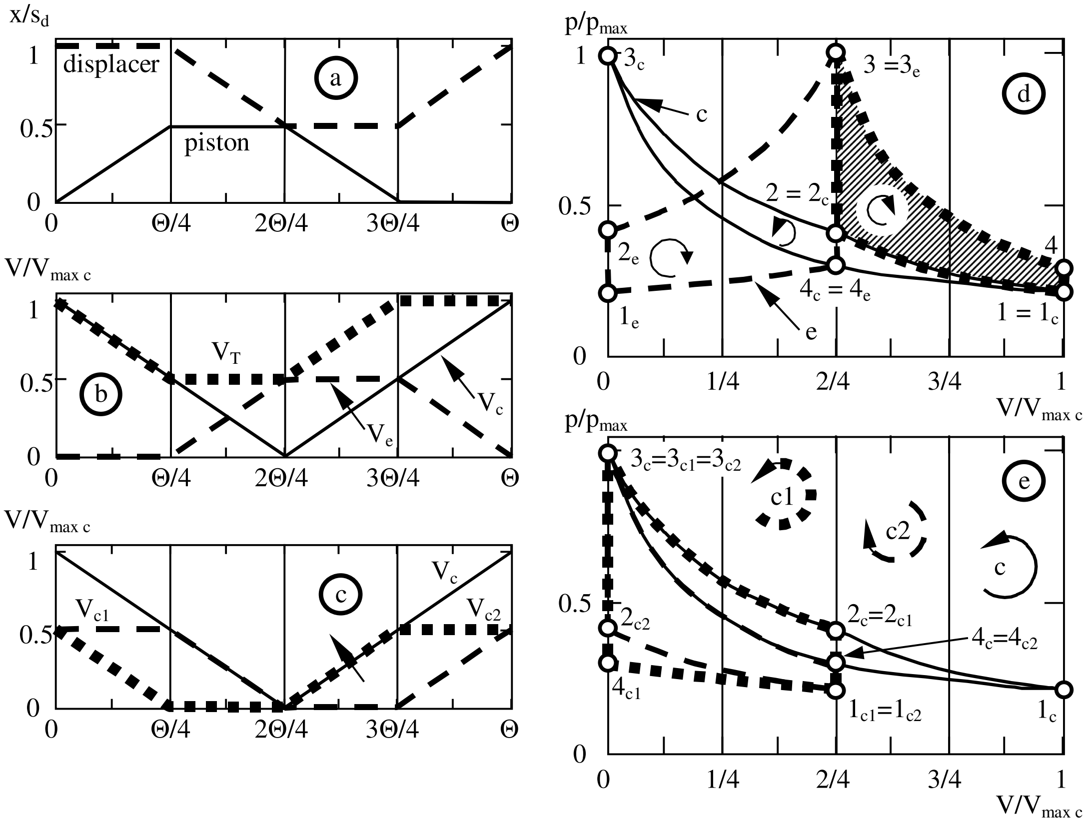

Fig. 4. Functioning of the theoretical gamma-type motor:
a - piston movement laws; b - volume variation laws; c - compression volumes variation laws; d - thermodynamic cycles inside the chambers and inside the motor; e - thermodynamic cycles inside the compression chambers c1 and c2; x = piston position; s_d = displacer stroke

Analyzing the processes inside the compression chambers c1 and c2, it follows that in the chamber c1 (fig. 4e) the gas evolves in a counterclockwise cycle, so this chamber spends work. The work spent is equal in module with the work exchanged in the expansion chamber, being transmitted from one chamber to another through the displacer. The equality of the two works appears because the pressure on the two sides of the displacer is the same and the volumes V_e and V_{c1} are phased with half of the cycle period Θ (the diameter of the piston stem is null).

It follows that the functional role of the two variable volume chambers inside the displacer cylinder is to obtain (due to the heat exchanged with the exterior, without any contribution of work) the pressure raise at constant volume V_M= const. and the pressure drop at V_m= const. (pressure variations compulsory for the functioning of the gamma-type machine as a motor).

The heat received by the motor in the expansion chamber (V_e) is transformed into work L_e. The work L_e is transmitted through the displacer to the chamber c1, where it is transformed again in heat. A fraction of the heat Q_{c1} is rejected from the motor in the chamber c1, and the rest of the heat is transformed in work in the chamber c2 (fig. 5). In the chamber c2 a clockwise cycle takes place, so here the motor produces the work exchanged with the exterior.

Inside the expansion chamber the following processes take place: the isothermal expansion (with variable mass m_T - the process 3_e-4_e on fig. 4d), the isothermal processes of gas exchange with the compression chamber (with variable mass, 2_e-3_e and 4_e-1_e) and the conventional process 1_e-2_e. The conventional process 1_e-2_e takes place with zero mass of gas and at zero volume. It corresponds to the process during which the gas inside the expansion chamber leaps from the state 1_e (characteristic to the end of the gas displacement into the compression chamber) to the state 2_e (characteristic to the beginning of the gas displacement back to the expansion chamber).

The processes inside the whole theoretical Stirling machine must be obtained by summing the processes that take place inside the compression and expansion chambers. During the compression process all the gas lies inside the compression chamber, so the constant mass isothermal process 1-2 is obtained.

The isochoric processes of gas transfer from one chamber to another (the processes 2-3 and 4-1) take place through the correlated variation of the chamber volumes, while the total volume occupied by the gas remains constant. In each chamber isothermal processes with variable mass take place, but inside the machine as a whole the temperature varies accordingly to the equation of the isochore.

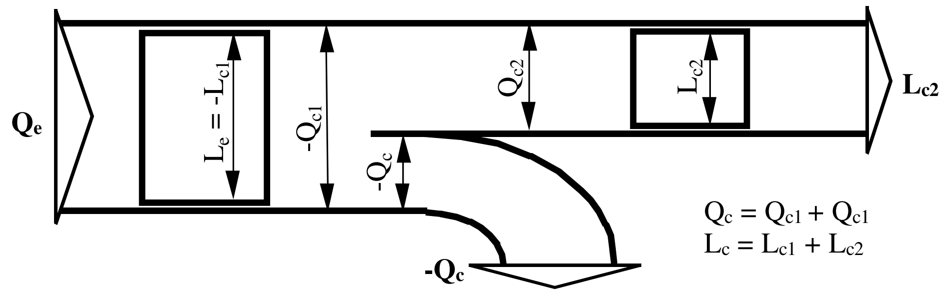

Fig. 5. Energy balance of the gamma-type machine

The works exchanged cyclically it the three chambers are calculated by integrating the defining relation dL = p dV. The pressure p is given by relation (1) and was particularized for each process in each functional chamber. The following relations are obtained:

$$L_e = m_T R \frac{T_c T_e}{T_e - T_c} \ln\left(\frac{T_c + T_e}{2T_c}\right); \qquad (2)$$

$$L_{c1} = -L_e; \qquad (3)$$

$$L_{c2} = m_T R T_c \ln\left(\frac{T_c + T_e}{2T_c}\right). \qquad (4)$$

In each isothermal chamber the work exchanged cyclically is equal with the heat exchanged. The thermal efficiency of the gamma-type motor is:

$$\eta_{t\gamma} = \frac{L_{c2}}{L_e} = 1 - \frac{T_c}{T_e} = \eta_{tCarnot}. \qquad (5)$$

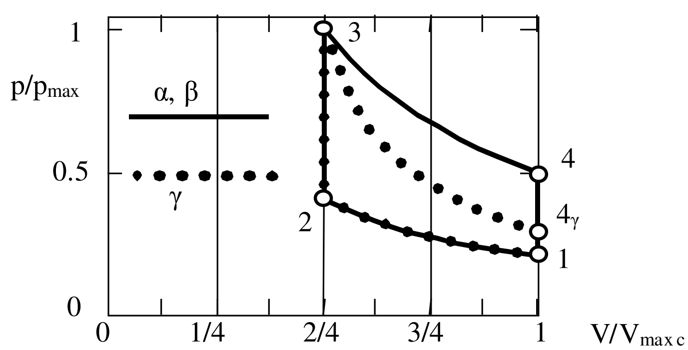

Fig. 6. p-V diagrams for theoretical alpha and beta-type Stirling motors and for a gamma-type motor

The thermal efficiency of the gamma-type motor is equal to the Carnot efficiency due to the isothermal processes in all the machine chambers and to the ideal regenerator.

In fig. 6 the p-V diagram of the gamma-type machine and the Stirling cycle that takes place between the same temperatures and volumes are presented. It follows that the work cyclically produced by the gamma-type motor is less than the one produced by a motor functioning in a Stirling thermodynamic cycle.

Conclusions

The paper presents an enlargement of the theoretical Stirling machine concept [2] at the theoretical gamma-type machine (currently named gamma-type Stirling machine [1], [3], [4]). The functioning model of the gamma-type machine (applied to a theoretical motor) considers that the two compression spaces are independent.

The p-V diagrams inside the three chambers of the motor and inside the machine as a whole show that the machine can realize only the isothermal compression process and the two isochoric processes required by the Stirling thermodynamic cycle. Instead of the isothermal expansion process of the Stirling cycle, inside the gamma-type machine the gas evolves in a process that is not a simple thermodynamic process (because in the expansion process the total mass of gas is placed in two chambers, at different temperatures).

Inside the compression chamber c1 the gas evolves in a counterclockwise cycle. Inside the expansion chamber and inside the compression chamber c2 the gas evolves in clockwise cycles. The work produced inside the expansion chambers is spent in the compression chamber c1. The work yielded to the exterior is produced inside the chamber c2. The thermal efficiency of the gamma-type motor is equal to the efficiency of a Carnot cycle that takes place between the same extreme temperatures. The work cyclically produced by the gamma-type motor is less than the work produced by a motor that works by a Stirling thermodynamic cycle.

As a final conclusion: inside a gamma-type machine the gas cannot evolve in a Stirling thermodynamic cycle, but in a cycle that eventually can be named "quasi-Stirling".

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Imposibilitatea modelării ciclului termodinamic Stirling pe maşini de tip constructiv gama

Rezumat

În lucrare este analizată maşina Stirling teoretică de tip gama. Se arată că pe această maşină nu se poate realiza ciclul termodinamic Stirling teoretic. S-a analizat funcţionarea maşinii şi fluxurile de energie schimbate cu exteriorul de camerele funcţionale. Se constată că randamentul maşinii izotermice de tip gama cu recuperarea căldurii este egal cu randamentul ciclului Carnot desfăşurat între aceleaşi temperaturi extreme. Lucrul mecanic produs este mai mic decât în cazul celorlalte maşini Stirling (pentru aceleaşi condiţii - volume, temperaturi, masă de agent).