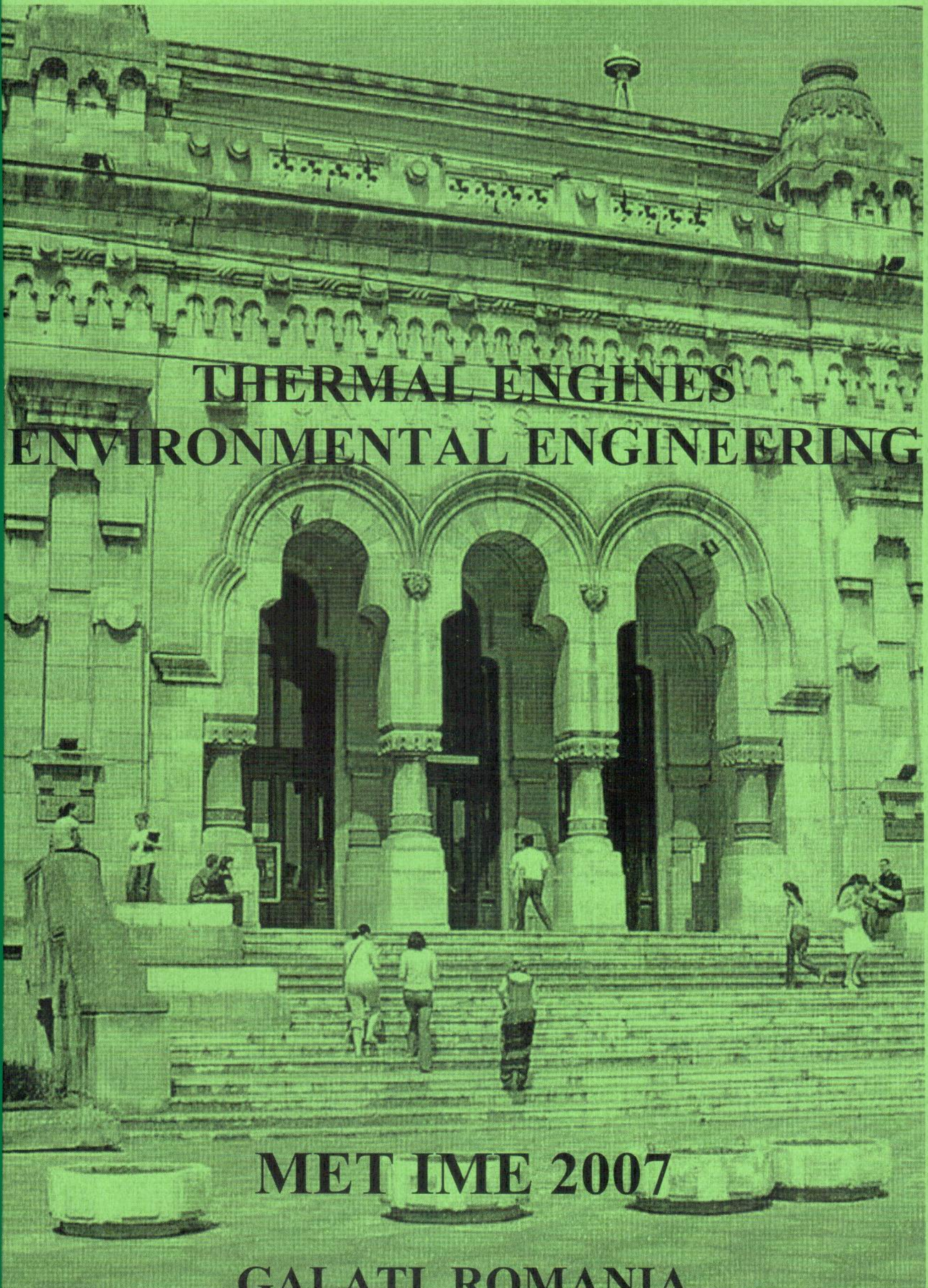

THERMAL ENGINES
ENVIRONMENTAL ENGINEERING

MET IME 2007

GALAȚI, ROMANIA

SECTION D
INTERNAL COMBUSTION ENGINES AND THEIR IMPACT ON THE ENVIRONMENT

SEMI-ADIABATIC PHYSICO-MATHEMATICAL MODEL FOR γ-TYPE STIRLING ENGINES

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ABSTRACT. *A physico-mathematical semi-adiabatic model for γ-type Stirling motors is presented. The model assumes that inside the expansion chamber and inside the compression space delimited by the displacer piston inside its cylinder the gas evolves in adiabatic processes. The differential equation system describing the functioning of the γ-type Stirling motor is obtained. A numerical example is used for comparison between the functioning of the motor accordingly to the proposed model and the isothermal functioning of the γ-type motor.*

KEYWORDS *γ-type motor, Stirling, semi-adiabatic model, compression space, displacer.*

1. INTRODUCTION

A functional unit of a γ-type Stirling engine comprises one displacer piston 2 and one power piston 9, each evolving in its own cylinder 3 and 10 (fig. 1).

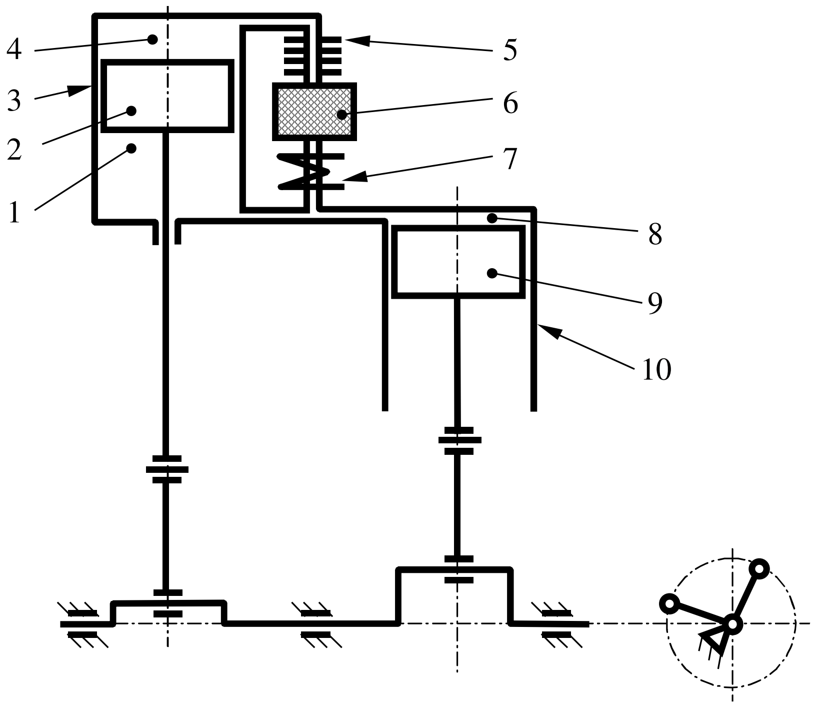

Fig. 1. Constructive scheme of a γ-type Stirling engine:
1 - compression space inside displacer's cylinder; 2 - displacer; 3 - displacer cylinder; 4 - expansion chamber; 5 - heater; 6 - regenerator; 7 - cooler; 8 - compression space inside power piston's cylinder; 9 - power piston; 10 - power piston's cylinder

The two pistons delimit two functional chambers with variable volume, one expansion chamber 4 and one compression chamber. The compression chamber is composed of two separate spaces 1 and 8, each space being placed inside another cylinder. The expansion and compression chambers are connected through three heat exchangers: heater 5, regenerator 6 and cooler 7.

The theoretical models of the Stirling engines assume that inside the variable volume chambers the working gas evolves in processes at constant temperature or in adiabatic processes. For the isothermal models - a case well studied in the literature [2], [4], [5] - in all the variable volume chambers only isothermal processes take place. The particular construction of the γ-type engine allows to elaborate some new theoretical physico-mathematical models assuming that adiabatic processes take place inside one or both of the compression spaces. In these cases the processes inside the expansion chamber are adiabatic.

2. SEMI-ADIABATIC PHYSICO-MATHEMATICAL MODEL

The semi-adiabatic model of the γ-type Stirling motor is based on the following hypotheses:

- the working agent is the ideal gas,
- the gas amount evolving inside the machine is constant,
- at thermodynamic level all cycle functional processes are time independent,
- the metallic parts of the machine (other than those of the power piston's cylinder walls confining the isothermal compression space) do not exchange heat either among them or with the exterior,
- the processes inside heat regenerators are ideal ones (regeneration efficiencies are 100%); the agent temperature inside the regenerator is deemed constant, being taken as logarithmic mean,
- inside the expansion chamber and inside the compression space inside the displacer's cylinder (fig. 2) adiabatic processes take place; so, the temperature inside these chambers vary cyclically,
- inside the compression space inside the power piston's cylinder (fig. 2) isothermal processes take place; so, the temperature inside this chamber is constant,
- inside the cooler and heater only isothermal processes take place,
- the instantaneous pressure is identical in all the spaces occupied by the agent, its value varying along the cycle,
- the movement of the displacer and of the power piston is the real movement, given by the crankshaft.

The hypotheses implying the temperatures inside the γ-type Stirling motor show that inside two of the machine chambers adiabatic processes take place and inside all other chambers isothermal processes take place only, thus confirming the described physico-mathematical model the denomination of semi-adiabatic model (this denomination is used also by West [6]).

To outline the semi-adiabatic character of the physico-mathematical model analyzed here, on fig. 2 the machine chambers are separate and placed in row. This presentation required the halving of the displacer. To each variable volume chamber inside the displacer's cylinder half of a displacer is assigned. The mechanical linkage between the displacer halves was symbolically drawn through bars that are exterior to the cylinder.

The following subscripts for dimensions inside machine chambers (volume V, temperature T, mass m) were used: e = expansion; h = heater; reg = regenerator; k = cooler; c = compression; 1 = displacer; 2 - power piston; T = total. The composed subscripts c1-k and h-e refer to the dimensions describing the separating sections between the adiabatic compression space and expansion chamber and their adjacent cooler and heater.

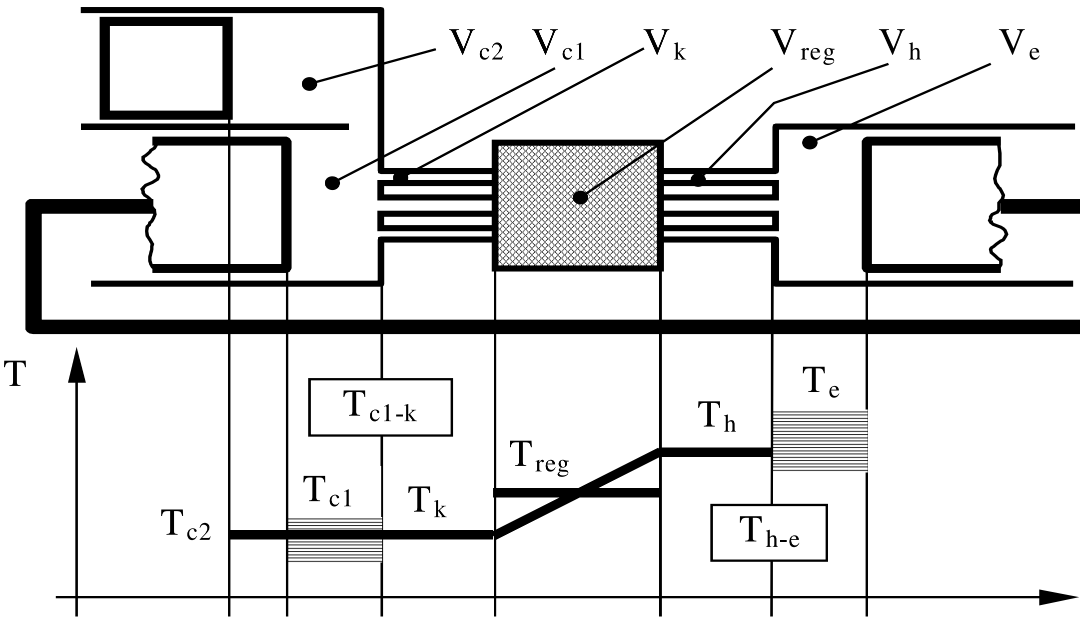

Fig. 2. Semi-adiabatic model of a γ-type Stirling motor

The model uses the differential equation of the conservation of the working agent total mass, the equation of state applied to the heat exchangers and to the isothermal compression space and the differential law of conservation of energy written for the adiabatic chambers.

The differential equation of the conservation of the working agent total mass is

$$d(m_T) = 0 = d(m_{c1}) + d(m_{c2}) + d(m_k) + d(m_{reg}) + d(m_h) + d(m_e)\,. \tag{1}$$

The differential expressions of the agent masses inside the heat exchangers are obtained from the equation of state, in which V = const. and T = const.:

$$\frac{dp}{p} = \frac{dm}{m}\,. \tag{2}$$

The mass m is taken from the equation of state and the differential expression of the mass inside a generic heat exchanger becomes:

$$dm_j = \frac{V_j}{R\,T_j}\,dp\,. \tag{3}$$

where the subscript j is replaced by h, reg and k.

For the isothermal compression space inside the power piston's cylinder the equation of state written at T = const. in differential form becomes:

$$\frac{dp}{p} + \frac{dV_{c2}}{V_{c2}} = \frac{dm_{c2}}{m_{c2}}\,. \tag{4}$$

For the mass differential expression the following form is obtained:

$$dm_{c2} = \frac{1}{R\,T_{c2}}(p\,dV_{c2} + V_{c2}\,dp)\,, \tag{5}$$

where $T_{c2} = T_k$.

Accordingly to the adopted hypotheses, inside the adiabatic chambers the gas exchanges work with the surrounding environment (through piston movement) and enthalpy with the neighbouring chambers (through the agent's entering the chamber from the neighbouring heater or cooler or leaving it toward the heater or cooler). The internal energy of the gas inside the adiabatic chamber changes as a consequence of mass and temperature variations. Inside these two adiabatic chambers the heat exchanged is zero, conformingly to the hypothesis of the model. The energy balance takes the expression:

$$dL + dU + dI = dQ\,. \tag{6}$$

For the adiabatic compression chamber (dQ = 0) the terms in (6) are explicated by the following relations:

$$dL_{c1} = p\,dV_{c1}\ , \tag{7}$$

$$dU_{c1} = d(c_v\,m_{c1}\,T_{c1}) = \frac{c_v}{R}(V_{c1}\,dp + p\,dV_{c1})\ , \tag{8}$$

$$dI_{c1-k} = d(c_p\,m_{c1-k}\,T_{c1-k}) = -c_p\,T_{c1-k}\,dm_{c1}\ . \tag{9}$$

Equation (9) takes into account that $dm_{c1\text{-}k} = -dm_{c1}$, because the mass of working agent that passes through the section c1-k is equal to the variation of the mass of the gas inside the space c1, taken with opposite sign. The positive sense of the agent flow inside the machine is considered to be from the compression chamber toward the expansion chamber. The term c_p m_{c1} $dT_{c1\text{-}k}$ was neglected, assuming the hypothesis according to which it is small in comparison with the other term.

Introducing the expressions (7), (8) and (9) in (6) and explaining the mass differential, the following relation is obtained for the differential of the working gas mass inside the adiabatic compression chamber:

$$dm_{c1} = \frac{1}{R\,T_{c1-k}}\left[p\,dV_{c1} + \frac{V_{c1}}{k}dp\right]. \tag{10}$$

Similarly, for the expansion chamber the next expression is obtained

$$dm_e = \frac{1}{R\,T_{h-e}}\left[p\,dV_e + \frac{V_e}{k}dp\right]. \tag{11}$$

Introducing the expressions of the mass differentials for all the chambers of the machine, given by (3), (5), (10) and (11) in (1), after some algebraic operations, the differential expression of the pressure is obtained:

$$dp = \frac{-k\,p\left[\dfrac{dV_{c1}}{T_{c1-k}}+\dfrac{dV_{c2}}{T_{c2}}+\dfrac{dV_e}{T_{h-e}}\right]}{\dfrac{V_e}{T_{h-e}}+\dfrac{V_{c1}}{T_{c1-k}}+k\left[\dfrac{V_{c2}}{T_{c2}}+\left(\dfrac{V_h}{T_h}+\dfrac{V_{reg}}{T_{reg}}+\dfrac{V_k}{T_k}\right)\right]}. \tag{12}$$

For the adiabatic chambers the differential expressions of the temperatures are taken from the equation of state. Particularizing for each adiabatic chamber, the following relations are obtained

$$dT_{c1} = T_{c1}\left(\frac{dp}{p}+\frac{dV_{c1}}{V_{c1}}-\frac{dm_{c1}}{m_{c1}}\right), \tag{13}$$

$$dT_e = T_e\left(\frac{dp}{p}+\frac{dV_e}{V_e}-\frac{dm_e}{m_e}\right). \tag{14}$$

Equations (12), (10), (11), (13) and (14) form the system of differential equations of the semi-adiabatic physico-mathematical model of the γ-type Stirling motor. The unknown functions are the pressure p, the masses m_{c1} and m_e inside the adiabatic compression and expansion chambers and the temperatures T_{c1} and T_e in the same chambers. The system is nonlinear, because there are several terms in the differential equations that have an order higher than one. The system has variable coefficients and the conditional temperatures T_{c1-k} and T_{h-e} of the agent that passes through the surfaces c1-k and h-e depend on the sense of the gas flow. The conditional temperatures take the expressions:

$$\begin{aligned}
&T_{c1-k} = T_{c1} \quad \text{if} \quad m_{c1-k} > 0 \quad (\text{or} \;\; dm_{c1} < 0);\\
&T_{c1-k} = T_k \quad \text{if} \quad m_{c1-k} < 0 \quad (\text{or} \;\; dm_{c1} > 0);\\
&T_{h-e} = T_h \quad \text{if} \quad m_{h-e} > 0 \quad (\text{or} \;\; dm_e > 0);\\
&T_{h-e} = T_e \quad \text{if} \quad m_{h-e} < 0 \quad (\text{or} \;\; dm_e < 0).
\end{aligned} \tag{16}$$

The system can be solved only by numerical integration. If the values of the unknown functions are adopted for a certain point in time, the problem is a initial value one and the numerical solution can be found with a Runge-Kutta method. The solution is obtained after several iterations, each of them using the previous one's results as initial values and thus getting closer to the result as the analysis goes on.

Inside the model the heat amounts cyclically exchanged inside the machine chambers are calculated for each chamber by integrating relation (6). The work cyclically exchanged inside each chamber is calculated by integrating the defining relation dL = p dV. The work L cyclically exchanged by the motor with the exterior is equal to the algebraic sum of heats exchanged inside the chambers.

The thermal efficiency of the γ-type Stirling motor is determined with the defining relation:

$$\eta_t = \frac{L}{Q_r}, \tag{17}$$

where Q_r is the heat cyclically received by the motor.

3. NUMERICAL EXAMPLE

A γ-type Stirling motor featuring the following dimensions is assumed: $D_1 = D_2 = 0{,}073$ m; $d_1 = 0{,}02$ m; $r_1 = r_2 = 0{,}0365$ m; $l_1 = l_2 = 0{,}15$ m; $f_{TDP1} = f_{BDP1} = f_{TDP2} = 0{,}001$ m; $V_h = V_k = 0.1\ V_S$; $V_{reg} = 1{,}2\ V_S$, where $V_S = 0.3055\ 10^{-3}\ m^3$ (volume swept by the displacer). Here D = cylinder diameter, d = stem diameter, r = crankshaft radius, l = rod lehght, f = dead space lenght and the subscript S stands for stroke. The machine works with a total mass of hydrogen m = 0.002 kg (R_{H2} = 4121 J/(kg K)) between temperatures T_h = 773 K and T_k = 310 K.

The numerical solving of the described semi-adiabatic model of the γ-type Stirling motor equations leads to the results displayed in fig. 3, fig. 4 and fig. 5, as well as inside table 1.

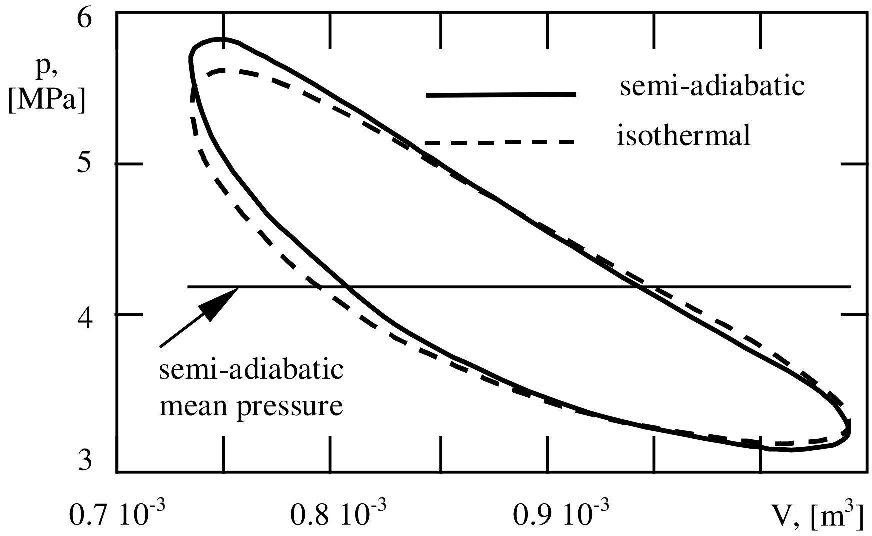

Fig. 3. Pressure variation inside the γ-type Stirling motor

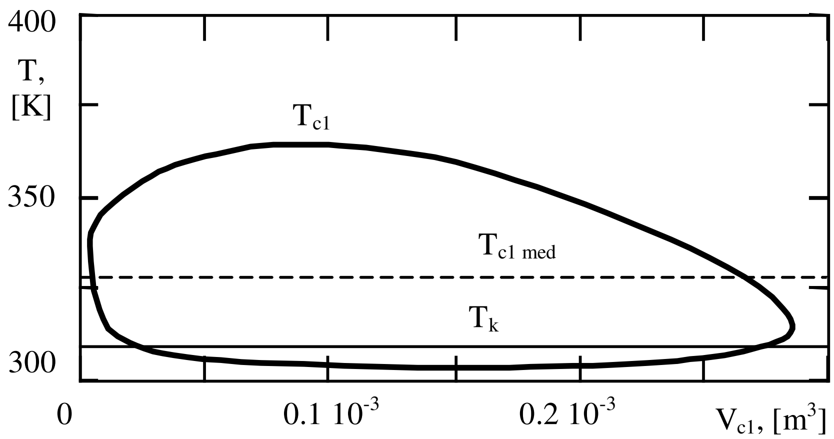

Fig. 4. Temperature variation inside the adiabatic compression chamber

The pressure variation inside the machine and the temperature variations in the adiabatic chambers are shown in fig. 3, fig. 4 and fig. 5.

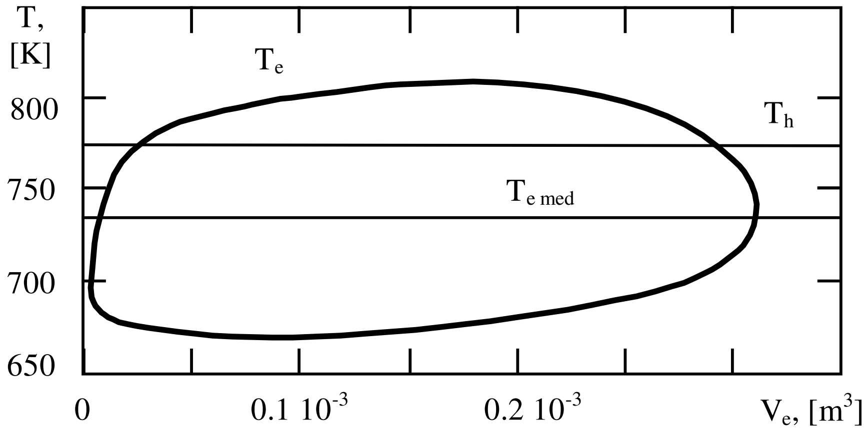

Fig. 5. Temperature variation inside the adiabaticexpansion chamber

Table 1. Numerical results

Model	Q_{c1}	Q_{c2}	Q_k	Q_h	Q_e	L_{c1}	L_{c2}	L_e	L	η_t
	[J/cycle]					[J/cycle]				-
isothermal	-456.6	258.6	0	0	493.7	-456.6	258.6	493.7	295.7	0.599
semi-adiabatic	0	241.1	-518.4	560.5	0	-518.3	241.1	560.4	283.1	0.505

4. CONCLUSIONS

The physico-mathematical semi-adiabatic model proposed for the numerical simulation of γ-type Stirling motor functioning allows for providing information on the possible performance the machine is capable of. Inside a real machine the heat exchanges do not take place isothermally, the heat regeneration is not ideal and the agent flow through the heat exchangers occurs with friction, all these facts lowering the performance beneath the semi-adiabatic one. The pressure variation inside the machine, calculated with the semi-adiabatic model, is close to the one calculated with the isothermal model, because a large quantity of the working agent is placed inside chambers considered to be isothermal in both models - especially in the regenerator.

In the semi-adiabatic model the temperature inside the adiabatic compression space is - for the most part of the cycle - above the neighbouring cooler temperature. The mean temperature inside this adiabatic chamber is above the cooler temperature too.

In the semi-adiabatic model the temperature inside the adiabatic expansion space is - for the most part of the cycle - below the neighbouring heater temperature. The mean temperature inside the expansion chamber is below the heater temperature too.

The model stresses the heat amounts exchanged inside the machine chambers. Accordingly to the adopted hypotheses, the cooler and the heater - that are adjacent to the adiabatic chambers - cyclically exchange nonzero heats. The isothermal compression space also exchanges heat with the low temperature source.

The heat amounts exchanged with the heat sources calculated using the semi-adiabatic model are larger than the corresponding ones calculated with the isothermal model. The work exchanged is greater for the isothermal model. The semi-adiabatic thermal efficiency is smaller than the isothermal one. The isothermal efficiency is equal to the thermal efficiency of a Carnot cycle evolving between the same temperatures. The semi-adiabatic model allows for a rapid analysis of the influence some constructive and functional factors have (more than the isothermal model can support) as well as for comparing different machines.

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