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OPTIMIZATION OF DIAMETER RATIO FOR THE VUILLEUMIER MACHINE BASED ON THE ISOTHERMAL FUNCTIONING

BY

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Abstract. A method for optimizing the diameter ratio for the Vuilleumier machine is presented. The optimum ratio giving maximum performances was determined with an isothermal functioning model. The optimum ratio depends on the heat exchanger volumes and on the stem diameter. If the stem diameter is null, the COP's are independent of diameter ratio.

Key words: Vuilleumier machine, thermal-acted, diameter optimization, isothermal model

1. Introduction

A Vuilleumier thermal-acted machine [1], [5], [6] is a machine inside which a constant amount of gas evolves inside an almost constant total volume. The gas lies inside several heat exchangers and four variable volume chambers placed inside two cylinders, each cylinder being fit with its own displacer piston. There are three levels of temperature inside the machine.

The refrigerating effect is acquired by expanding the gas inside a low temperature chamber. Pressure variation inside the machine is acquired by heating the agent inside a high temperature chamber and by cooling the agent inside two intermediate temperature chambers.

According to the schematic diagram in fig. 1, a Vuilleumier machine is comprised of a cold cylinder 1 and a hot cylinder 15 inside which the cold displacer 3 and the hot displacer 13 work. The cold cylinder and displacer share a diameter inferior to the one shared by the hot cylinder and displacer. A drive comprised of crankshaft 18 and rods 17 and 19 provide movement for the displacers. The cold displacer splits the space inside its cylinder in two chambers: a low temperature one 4 and an intermediate temperature one 2.

Inside the hot cylinder the hot displacer delimits a high temperature chamber 12 and an intermediate temperature chamber 14. Each cylinder is fit with its own heat exchanger set. The cold cylinder has a low temperature heater 5, a low temperature regenerator 6 and an intermediate temperature cooler 7. The hot cylinder is fit with an intermediate temperature cooler 9, a hot temperature regenerator 10 and a high temperature heater 11. The intermediate temperature cooling chambers are connected through pipe 8.

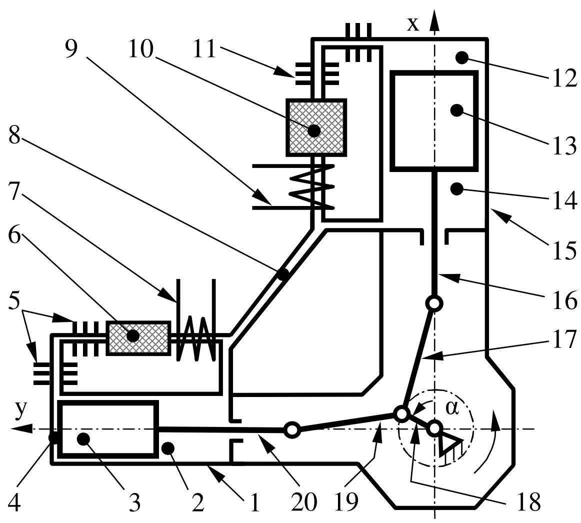

Fig. 1 – Vuilleumier thermal-acted heat pump:
and 15 - cylinder; 2 and 14 - intermediate temperature chamber; 3 and 13 - displacer; 4 - low temperature chamber; 5 - low temperature heater; 6 and 10 - regenerator; 7 and 9 - cooler; 8 - connection pipe; 11 - high temperature heater; 12 - high temperature chamber; 16 and 20 - stem; 17 and 19 - rod; 18 - crankshaft.

2. Vuilleumier Machine Performances calculated with an Isothermal Model

The isothermal physico-mathematical [2] model is based on the following hypotheses:

- the working agent is the ideal gas;
- the gas amount evolving inside the machine is constant;
- at thermodynamic level all cycle functional processes are time independent;
- the metallic parts of the machine (other than heat exchangers and cylinder walls) do not exchange heat either among them or with the exterior;
- the processes inside heat regenerators are ideal ones (regeneration efficiencies are 100%); the agent temperature inside the regenerator is deemed constant, being taken as either logarithmic or arithmetic mean;
- the agent temperature inside the low temperature chamber is equal to the

one inside the low temperature heater, the one of the outer heating agent, the one of the cylinder walls and the one of the cold displacer frontal surface;

● the agent temperature inside the high temperature chamber is equal to the one inside the high temperature heater, the one of the outer heating agent, the one of the cylinder walls and the one of the hot displacer frontal surface;

● the agent temperature inside the intermediate temperature chambers is equal to the one inside the coolers, the one of the outer cooling agent, the one of the cylinder walls next to those respective chambers and the one of the stems and of the displacer bottoms;

● the instantaneous pressure is identical in all the spaces occupied by the agent, its value varying along the cycle;

● the volume variation law for the chambers inside the machine cylinders is known.

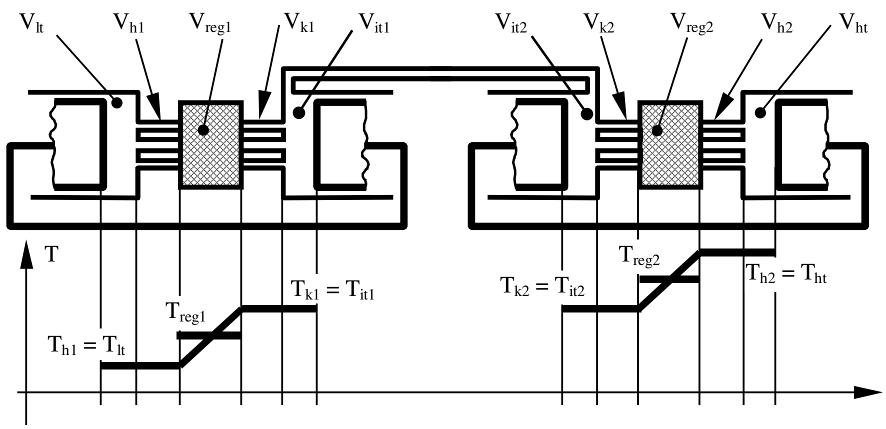

Fig. 2 – Isothermal model of the Vuilleumier machine:

The hypotheses implying the temperatures inside the Vuilleumier machine show that inside all chambers isothermal processes take place only. To outline the isothermal character of the physico-mathematical model, on fig. 2 the machine chambers are separate and placed in row. This presentation required the halving of each displacer. Each variable volume chamber is assigned half a displacer. The mechanical linkage between the displacer halves was symbolically drawn through bars exterior to the cylinder.

We used the following subscripts for dimensions inside machine chambers (volume V, temperature T, mass m): h = heater; reg = regenerator; k = cooler; 1 = cold displacer; 2 - hot displacer; ht = high temperature; lt = low temperature; it = intermediate temperature.

The pressure variation law p(α) was determined from the equation of conservation for the total gas mass inside the machine:

(1) $$m=\sum_i \frac{p(\alpha)\,V_i(\alpha)}{R\,T_i},$$

where the masses m_i are calculated from the equation of state for each chamber.

The pressure acquires the expression:

(2) $$p(\alpha)=\frac{m\,R}{\sum_i \frac{V_i(\alpha)}{T_i}}.$$

Temperature values T_{lt} = T_{h1}, T_{k1} = T_{k2} = T_{it1} = T_{it2}, T_{ht} = T_{h2} are imposed and the chamber volumes variation laws are imposed by the geometry chosen for the machine. Regenerator mean temperatures are calculated with relations:

(3) $$T_{reg1}=\frac{T_c-T_{lt}}{\ln(T_c/T_{lt})} \text{ and } T_{reg2}=\frac{T_{ht}-T_c}{\ln(T_{ht}/T_c)}.$$

Accordingly to the first law of thermodynamics applied to the agent undergoing a cycle inside an isothermal chamber, the heat exchanged is equal to the work exchanged and is calculated with the relations:

(4) $$Q_{lt}=\int_0^{2\pi} p(\alpha)\left[\frac{dV_{lt}(\alpha)}{d\alpha}\right]d\alpha;\quad Q_{it1}=\int_0^{2\pi} p(\alpha)\left[\frac{dV_{it1}(\alpha)}{d\alpha}\right]d\alpha;$$

(5) $$Q_{it2}=\int_0^{2\pi} p(\alpha)\left[\frac{dV_{it2}(\alpha)}{d\alpha}\right]d\alpha;\quad Q_{ht}=\int_0^{2\pi} p(\alpha)\left[\frac{dV_{ht}(\alpha)}{d\alpha}\right]d\alpha.$$

As a consequence of the displacer stem's presence, the machine exchanges with the environment an amount of work per cycle having the expression

(6) $$L=Q_{lt}+Q_{it1}+Q_{it2}+Q_{ht}.$$

The useful effect of the Vuilleumier refrigerating machine is represented by heat Q_{lt} extracted from the low temperature heat source and the useful effect of the heat pump is heat (Q_{it1} +Q_{it2}) transmitted to the user at an intermediate temperature of T_k.

The Vuilleumier refrigerating machine and the Vuilleumier heat pump are characterized by the following coefficients of performance:

(7) $$\varepsilon_r = Q_{lt} / Q_{ht} \quad \text{and} \quad \varepsilon_{hp} = \left| Q_{it1} + Q_{it2} \right| / Q_{ht} .$$

The performances of the Vuilleumier machine can also be calculated using another physico-mathematical functioning model, as the semi-adiabatic [3] or adiabatic [4] models.

3. Optimization of the Diameters Ratio

The paper analyzes a Vuilleumier machine for which we know:
- the working agent (constant R);
- the temperatures of the three heat sources;
- the dimensions of the motion mechanism (identical for both displacers that as a consequence share the same stroke);
- the volumes of the heat exchangers, calculated as ratios of the maximum volume of the high temperature chamber;
- the sum between the maximum volume of the high temperature chamber and the maximum volume of the low temperature chamber;
- the stem diameters;
- the pressure of the working agent inside the machine at ambient temperature (that establishes the agent mass).

We use the volume ratio:

(8) $$\varepsilon_v = \frac{V_{lt\,max}}{V_{ht\,max}} = \left(\frac{d_1}{d_2} \right)^2 = (\varepsilon_d)^2 .$$

where d is diameter and ε_d is the diameter ratio. Subscript “max” refers to the maximum value.

For optimization the values ε_d for which each of the machine performances is maximum must be calculated:
- the heat Q_{lt} taken from the low temperature heat source for the refrigerating Vuilleumier machine;
- the heat $Q_{it1} + Q_{it2}$ transmitted to intermediate temperature heat source for the Vuilleumier heat pump;
- the machine COP.

At the optimization analysis the sum $V_{lt\,max} + V_{ht\,max}$ between the maximum volume of the high temperature chamber and the maximum volume of the low temperature chamber was considered constant. This condition means that the sum of the four variable volumes of the machine does not depend on ratio ε_v but only on the current position of the crankshaft.

The numerical modeling of the isothermal machine was used. Of course, both d_1 and d_2 must be greater than d_{stem}. All machine performances must be represented as functions of ε_d. The maximum values of these functions were considered as optimums. Because the performances depend on the heat exchanger volumes also, the optimum values depend on these volumes too. This observation permits to iterate the calculations for the heat exchanger volumes determined for the optimum ε_d ratio. The process is a quickly convergent one.

4. Numerical Example

A Vuilleumier machine featuring the following dimensions is assumed: d_1= 0.1 m; $d_{stem\,1}$= $d_{stem\,2}$ = 0.02 m; d_2 = 0.12 m; $r_1 = r_2$ = 0.05 m; $l_1 = l_2$ = 0.2 m; $f_{TDP1} = f_{BDP1} = f_{TDP2} = f_{BDP2}$ = 0.001 m; $V_{h1} = V_{h2} = V_{k1} = V_{k2} = 0.05\ V_{ht\ max}$; $V_{reg1} = V_{reg2} = 1.2\ V_{ht\ max}$, where $V_{ht\ max}$ = maximum volume of the high temperature chamber, d = cylinder diameter, d_{stem} = stem diameter, r = crankshaft radius, l = rod length, f = dead space length. For the chosen Vuilleumier machine $V_{ht\ max} + V_{lt\ max} = 1.936\ 10^{-3}\ m^3$ (a constant value for the optimization calculations). The machine works with a total mass of hydrogen m (corresponding to a pressure of 50 bar in the machine, at an ambient temperature of 15 ºC; R_{H2} = 4121 J/(kg K)) between temperatures $T_{ht} = T_{h2}$ = 923 K; $T_{k1} = T_{k2} = T_{it1} = T_{it2}$ = 343 K and $T_{lt} = T_{h1}$ = 278 K.

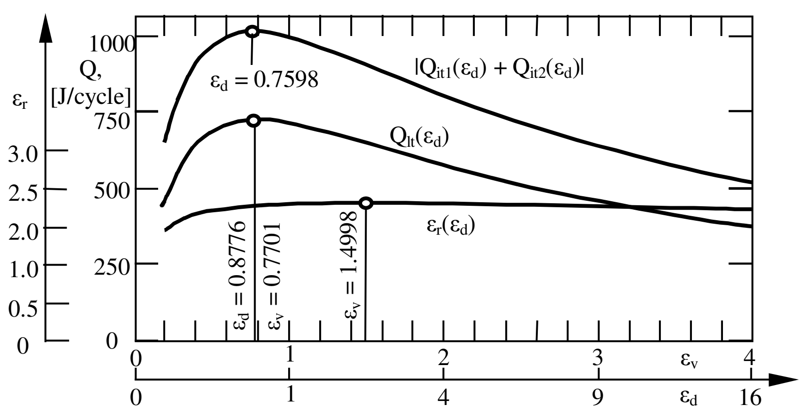

Fig. 3 – Variation of the performances on ε_d and ε_v ratios

The most important numerical results obtained for a Vuilleumier refrigeration machine are presented in Fig. 3, Fig. 4 and Fig. 5. The variations of the terms Q_{lt}, $|Q_{it1} + Q_{it2}|$ and ε_r as functions of ε_d and ε_v ratios are presented in Fig. 3. All three functions reach their maximum values.

In Fig. 4 the values Q_{lt} and ε_r were divided by their maximum values. This operation evidenced the ε_d interval for which the performances are less than the maximum ones with a chosen difference (5% on Fig. 3).

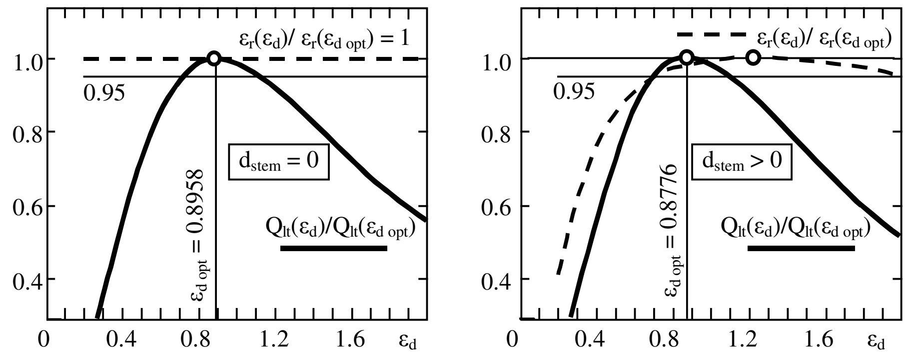

Fig. 4 – Optimum diameter ratios for a Vuilleumier refrigeration machine:
a – for $d_{stem} = 0$; *b* – for $d_{stem} > 0$

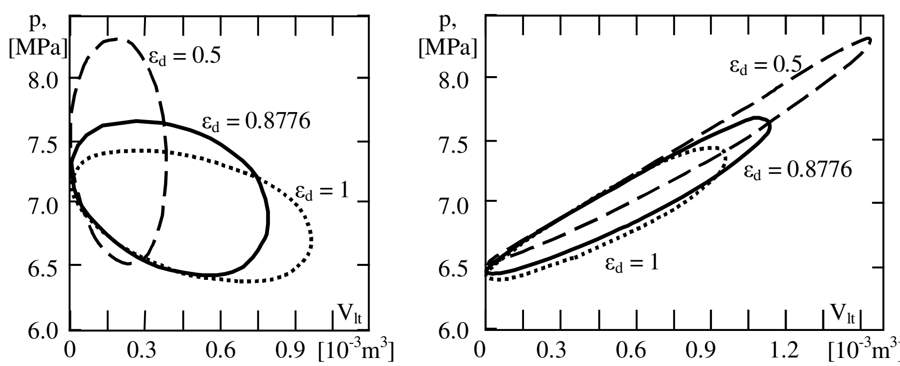

Fig. 5 – Indicator diagrams inside low and high temperature chambers for different ε_d values:
a – inside low temperature chamber; *b* – inside high temperature chamber

The influence of ε_d ratio over the functioning of the Vuilleumier machine was evidenced using the indicator diagrams p(V). In Fig. 5, besides the diagrams inside low and high temperature chambers (lt and ht) for the optimum value ε_d, diagrams for another two values were represented.

5. Conclusions

1. For a Vuilleumier machine having $d_{stem} = 0$ the COP's ε_r and ε_{hp} do not depend on ε_d ratio.

2. The COP's depend on ε_d when $d_{stem} > 0$, because the work exchanged is higher than zero.

3. The optimum values of ε_v or ε_d ratios depend on the heat exchanger volumes and on the stem diameters.

4. The proposed optimization method permits to establish the diameter d_1 and d_2 of the cylinders for the maximum performances. The optimum value for a Vuilleumier refrigerator is different from the optimum value for the heat pump.

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Optimizarea raportului diametrelor cilindrilor maşinii Vuilleumier pe baza modelului de funcţionare izotermic

(Rezumat)

Se prezintă o metodă de optimizare a raportului diametrelor cilindrilor maşinii Vuilleumier pentru obţinerea unor performanţe maxime. Valoarea optimă a raportului diametrelor determinată prin simulare numerică a funcţionării maşinii cu metoda izotermică depinde de volumele schimbătoarelor de căldură şi de diametrul tijelor împingătoarelor. Dacă diametrul tijei este zero, eficienţa frigorifică nu depinde de raportul diametrelor, spre deosebire de căldurile schimbate în camerele maşinii. Există o plajă largă de rapoarte ale diametrelor pentru care performanţele maşinii Vuilleumier se menţin la valori ridicate, apropiate de valorile maxime.